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Modal characteristics of structures considering dynamic soil-structure interaction effects

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ABSTRACT

The modal characteristics of structures are usually computed disregarding any interaction with the soil. This paper presents a finite element-perfectly matched layers model to compute the modal characteristics of 2D and 3D coupled soil-structure systems while taking fully into account dynamic soil-structure interaction. The methodology can facilitate the interpretation of experimentally identified modal characteristics by assessing the importance of dynamic soil-structure interaction.

1. Introduction

The modal characteristics of structures are usually computed with finite element models disregarding any interaction with the soil. These modal characteristics can differ from the ones identified by means of experimental modal analysis [1]. Finite element updating is used to reduce the discrepancy between numerically predicted and experimentally identified modal characteristics by appropriately calibrating model parameters [2]. Dynamic soil-structure interaction (SSI) affects the modal characteristics due to the more flexible support conditions and the dissipation of energy in the soil [3]. Disregarding dynamic SSI might result in poor correspondence between numerical and experimental modal characteristics. Effects from dynamic SSI might be erroneously lumped to structural parameters during finite element updating, leading to model errors adversely affecting accurate prediction of structural vibration. Dynamic SSI can be accounted for by using coupled finite element-boundary element (FE-BE) formulations [4] or finite element formulations in conjunction with absorbing boundary conditions (ABC) [5] or perfectly matched layers (PML) [6]. In these models, the influence of the semi-infinite extent of the soil is explicitly taken into account by allowing the radiation of elastodynamic waves.

The computation of the modal characteristics of these coupled soil-structure models requires the solution of non-linear eigenvalue problems which are more challenging to solve than the generalized eigenvalue problem. This paper presents a FE-PML model facilitating the computation of the modal characteristics of 2D and 3D coupled soil-structure systems. These results can support the interpretation of experimentally identified modal characteristics by quantifying the influence of dynamic SSI. Ultimately, the FE-PML model can be used in vibration based finite element updating where both soil and structural parameters are calibrated.

2. FE-PML model

Fig. 1 shows the FE-PML model used to compute the modal characteristics of coupled soil-structure systems. The structure Ω_{b} is partially embedded in a stratified soil $\Omega_{\mathrm{s}}^{\mathrm{e}}$. The computational domain Ω is composed by the generalized structure $\Omega_{\mathrm{r}} = \Omega_{\mathrm{b}} \cup \overline{\Omega}_{\mathrm{s}}^{\mathrm{e}}$ modeled with FE and the PML buffer zone Ω_{p} simulating the truncated unbounded soil at Σ_{rp}.

The virtual work equation for the generalized structure Ω_{r} in the frequency domain is:

$$\int_{\Omega_{\mathrm{r}}} (\mathbf{L}\hat{\mathbf{v}})^{\mathrm{T}}\mathbf{C}(\mathbf{L}\hat{\mathbf{u}})\mathrm{d}\Omega + (\mathrm{i}\omega)^2 \int_{\Omega_{\mathrm{r}}} \rho \hat{\mathbf{v}}^{\mathrm{T}}\hat{\mathbf{u}}\,\mathrm{d}\Omega = \int_{\Gamma_{\mathrm{r}}^{\mathrm{N}}} \hat{\mathbf{v}}^{\mathrm{T}}\hat{\mathbf{t}}^{\mathbf{n}}\,\mathrm{d}\Gamma + \int_{\Sigma_{\mathrm{rp}}} \hat{\mathbf{v}}^{\mathrm{T}}\hat{\mathbf{t}}^{\mathbf{n}}\,\mathrm{d}\Gamma \tag{1}$$

where $\hat{\mathbf{u}}$ is the displacement vector, $\hat{\boldsymbol{\epsilon}} = \{\hat{\epsilon}_{xx}, \hat{\epsilon}_{yy}, \hat{\epsilon}_{zz}, \hat{\gamma}_{xy}, \hat{\gamma}_{yz}, \hat{\gamma}_{zx}\}^{\mathrm{T}} = \mathbf{L}\hat{\mathbf{u}}$ is the strain vector, $\mathbf{L}$ is a matrix containing differential operators, $\hat{\boldsymbol{\sigma}} = \{\hat{\sigma}_{xx}, \hat{\sigma}_{yy}, \hat{\sigma}_{zz}, \hat{\sigma}_{xy}, \hat{\sigma}_{yz}, \hat{\sigma}_{zx}\}^{\mathrm{T}} = \mathbf{C}\hat{\boldsymbol{\epsilon}}$ is the stress vector collecting the elements of the symmetric stress tensor σ_{ij}, $\mathbf{C}$ is the constitutive matrix, ρ is the density, $\hat{\mathbf{t}}^{\mathbf{n}}$ are applied tractions with $\mathbf{n}$ the unit outward normal vector and $\hat{\mathbf{v}}$ is a kinematically admissible virtual displacement field on Ω. A hat above a variable denotes its representation in the frequency

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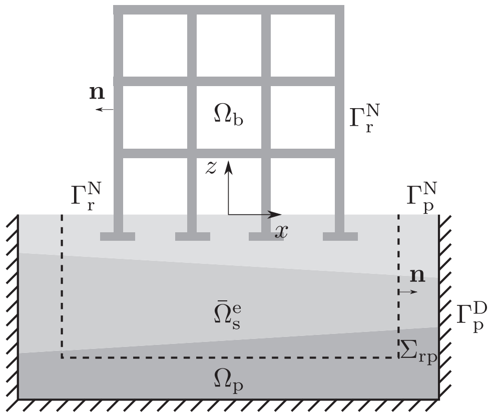

Fig. 1. FE-PML model.

domain. The last integral on the right hand-side is the interaction term on Σ_{rp} with the PML buffer zone Ω_{p} where the traction equilibrium $\hat{\mathbf{t}}_{\mathrm{r}}^{\mathbf{n}} + \hat{\mathbf{t}}_{\mathrm{p}}^{-\mathbf{n}} = \mathbf{0}$ holds.

Complex coordinate stretching is applied inside the PML buffer zone Ω_{p} in order to artificially attenuate the elastodynamic waves [6,7]. For a coordinate s, representing the x, y or z coordinate, the stretched coordinate $\tilde{s}$ is defined as:

$$\tilde{s} = s_{\mathrm{o}} + \int_{s_{\mathrm{o}}}^{s_{\mathrm{t}}} \hat{\lambda}_{\mathrm{s}}(s)\mathrm{d}s = s_{\mathrm{o}} + \int_{s_{\mathrm{o}}}^{s_{\mathrm{t}}} \alpha_{0\mathrm{s}}(s)\mathrm{d}s + \frac{1}{\mathrm{i}\omega}\int_{s_{\mathrm{o}}}^{s_{\mathrm{t}}} \alpha_{1\mathrm{s}}(s)\mathrm{d}s \tag{2}$$

where s_{o} and s_{t} delimit the origin and the termination of the PML buffer zone in the direction of the coordinate s and $\hat{\lambda}_{\mathrm{s}}(s)$ is the stretch function with $\alpha_{0\mathrm{s}}(s)$ and $\alpha_{1\mathrm{s}}(s)$ polynomial functions controlling the attenuation of the evanescent and propagating waves inside the PML buffer zone [8]. Introducing the complex coordinate stretching (2), the equilibrium equation of the PML buffer zone Ω_{p} is:

$$(\hat{\lambda}_y\hat{\lambda}_z\mathbf{L}_x^{\mathrm{T}} + \hat{\lambda}_x\hat{\lambda}_z\mathbf{L}_y^{\mathrm{T}} + \hat{\lambda}_x\hat{\lambda}_y\mathbf{L}_z^{\mathrm{T}})\hat{\boldsymbol{\sigma}} = (\mathrm{i}\omega)^2\rho\hat{\lambda}_x\hat{\lambda}_y\hat{\lambda}_z\hat{\mathbf{u}} \quad \text{in } \Omega_{\mathrm{p}} \tag{3}$$

where the differential operators $\mathbf{L}_x$, $\mathbf{L}_y$ and $\mathbf{L}_z$ are defined as:

$$\mathbf{L}_x = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial}{\partial x} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\partial}{\partial x} \end{bmatrix}^{\mathrm{T}}$$

$$\mathbf{L}_y = \begin{bmatrix} 0 & 0 & 0 & \frac{\partial}{\partial y} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial}{\partial y} & 0 \end{bmatrix}^{\mathrm{T}}$$

$$\mathbf{L}_z = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \frac{\partial}{\partial z} \\ 0 & 0 & 0 & 0 & \frac{\partial}{\partial z} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}} \tag{4}$$

Similarly, the kinematic equation of the PML buffer zone in stretched coordinates, using $\hat{\boldsymbol{\varepsilon}} = \mathbf{D}\hat{\boldsymbol{\sigma}}$ with $\mathbf{D}$ the compliance matrix, is:

$$\hat{\lambda}_x\hat{\lambda}_y\hat{\lambda}_z\mathbf{D}\hat{\boldsymbol{\sigma}} = (\hat{\lambda}_y\hat{\lambda}_z\mathbf{L}_x + \hat{\lambda}_x\hat{\lambda}_z\mathbf{L}_y + \hat{\lambda}_x\hat{\lambda}_y\mathbf{L}_z)\hat{\mathbf{u}} \quad \text{in } \Omega_{\mathrm{p}} \tag{5}$$

The mixed formulation of Fathi et al. [9] is used for the modeling of the PML buffer zone Ω_{p} where both displacements and stresses are retained as independent variables. The equilibrium Eq. (3) and the kinematic Eq. (5) are treated independently. The integral form of the equilibrium Eq. (3) is obtained by considering a kinematically admissible virtual displacement field $\hat{\mathbf{v}}$ on Ω, integrating by parts the terms depending on $\hat{\boldsymbol{\sigma}}$ and applying the divergence theorem:

$$\int_{\Omega_{\mathrm{p}}} (\hat{\lambda}_y\hat{\lambda}_z\mathbf{L}_x\hat{\mathbf{v}} + \hat{\lambda}_x\hat{\lambda}_z\mathbf{L}_y\hat{\mathbf{v}} + \hat{\lambda}_x\hat{\lambda}_y\mathbf{L}_z\hat{\mathbf{v}})^{\mathrm{T}}\hat{\boldsymbol{\sigma}}\,\mathrm{d}\Omega + (\mathrm{i}\omega)^2\int_{\Omega_{\mathrm{p}}} \rho\hat{\lambda}_x\hat{\lambda}_y\hat{\lambda}_z\hat{\mathbf{v}}^{\mathrm{T}}\hat{\mathbf{u}}\,\mathrm{d}\Omega = \int_{\Sigma_{\mathrm{rp}}} \hat{\mathbf{v}}^{\mathrm{T}}\hat{\mathbf{t}}^{-\mathbf{n}}\,\mathrm{d}\Gamma \tag{6}$$

where the integral on the right hand-side is the interaction term with the generalized structure Ω_{r}. The integral form of the kinematic Eq. (5) is obtained by considering a virtual stress field $\hat{\boldsymbol{\tau}}$ on Ω:

$$\int_{\Omega_{\mathrm{p}}} \hat{\boldsymbol{\tau}}^{\mathrm{T}}(\hat{\lambda}_y\hat{\lambda}_z\mathbf{L}_x + \hat{\lambda}_x\hat{\lambda}_z\mathbf{L}_y + \hat{\lambda}_x\hat{\lambda}_y\mathbf{L}_z)\hat{\mathbf{u}}\,\mathrm{d}\Omega - \int_{\Omega_{\mathrm{p}}} \hat{\lambda}_x\hat{\lambda}_y\hat{\lambda}_z\hat{\boldsymbol{\tau}}^{\mathrm{T}}\mathbf{D}\hat{\boldsymbol{\sigma}}\,\mathrm{d}\Omega = 0 \tag{7}$$

The dynamic SSI problem is formulated by taking into account the equilibrium of tractions on the interface Σ_{rp}. Adding Eqs. (1) and (6) yields:

$$\int_{\Omega_{\mathrm{r}}} (\mathbf{L}\hat{\mathbf{v}})^{\mathrm{T}}\mathbf{C}(\mathbf{L}\hat{\mathbf{u}})\mathrm{d}\Omega + (\mathrm{i}\omega)^2\int_{\Omega_{\mathrm{r}}} \rho\hat{\mathbf{v}}^{\mathrm{T}}\hat{\mathbf{u}}\,\mathrm{d}\Omega + \int_{\Omega_{\mathrm{p}}} (\hat{\lambda}_y\hat{\lambda}_z\mathbf{L}_x\hat{\mathbf{v}} + \hat{\lambda}_x\hat{\lambda}_z\mathbf{L}_y\hat{\mathbf{v}} + \hat{\lambda}_x\hat{\lambda}_y\mathbf{L}_z\hat{\mathbf{v}})^{\mathrm{T}}\hat{\boldsymbol{\sigma}}\,\mathrm{d}\Omega + (\mathrm{i}\omega)^2\int_{\Omega_{\mathrm{p}}} \rho\hat{\lambda}_x\hat{\lambda}_y\hat{\lambda}_z\hat{\mathbf{v}}^{\mathrm{T}}\hat{\mathbf{u}}\,\mathrm{d}\Omega = \int_{\Gamma_{\mathrm{r}}^{\mathrm{N}}} \hat{\mathbf{v}}^{\mathrm{T}}\hat{\mathbf{t}}^{\mathbf{n}}\,\mathrm{d}\Gamma \tag{8}$$

The combined integral Eqs. (7) and (8) describe the dynamic response of the coupled soil-structure system. A standard Galerkin procedure is followed in the FE implementation. The displacement field $\hat{\mathbf{u}}$ and the virtual displacement field $\hat{\mathbf{v}}$ are approximated as $\hat{\mathbf{u}} \simeq \mathbf{N}_{\mathrm{u}}\underline{\hat{\mathbf{u}}}$ and $\hat{\mathbf{v}} \simeq \mathbf{N}_{\mathrm{u}}\underline{\hat{\mathbf{v}}}$ with $\mathbf{N}_{\mathrm{u}}$ a matrix containing globally defined shape functions. Similarly, the stress field $\hat{\boldsymbol{\sigma}}$ and the virtual stress field $\hat{\boldsymbol{\tau}}$ are approximated as $\hat{\boldsymbol{\sigma}} \simeq \mathbf{N}_{\sigma}\underline{\hat{\boldsymbol{\sigma}}}$ and $\hat{\boldsymbol{\tau}} \simeq \mathbf{N}_{\sigma}\underline{\hat{\boldsymbol{\tau}}}$. Since Eqs. (7) and (8) hold for any kinematically admissible virtual displacement field $\hat{\mathbf{v}}$ and virtual stress field $\hat{\boldsymbol{\tau}}$, the following system of equations is obtained:

$$\begin{bmatrix} \hat{\mathbf{S}}_{\mathrm{uu}} & \hat{\mathbf{S}}_{\mathrm{u}\sigma} \\ \hat{\mathbf{S}}_{\mathrm{u}\sigma}^{\mathrm{T}} & \hat{\mathbf{S}}_{\sigma\sigma} \end{bmatrix} \begin{Bmatrix} \underline{\hat{\mathbf{u}}} \\ \underline{\hat{\boldsymbol{\sigma}}} \end{Bmatrix} = \begin{Bmatrix} \underline{\hat{\mathbf{p}}} \\ \mathbf{0} \end{Bmatrix} = \begin{Bmatrix} \int_{\Gamma_{\mathrm{r}}^{\mathrm{N}}} \mathbf{N}_{\mathrm{u}}^{\mathrm{T}}\hat{\mathbf{t}}^{\mathbf{n}}\mathrm{d}\Gamma \\ \mathbf{0} \end{Bmatrix} \tag{9}$$

where the block matrices are defined as follows:

$$\hat{\mathbf{S}}_{\mathrm{uu}} = \int_{\Omega_{\mathrm{r}}} (\mathbf{L}\mathbf{N}_{\mathrm{u}})^{\mathrm{T}}\mathbf{C}(\mathbf{L}\mathbf{N}_{\mathrm{u}})\mathrm{d}\Omega + (\mathrm{i}\omega)^2\int_{\Omega_{\mathrm{r}}} \rho\mathbf{N}_{\mathrm{u}}^{\mathrm{T}}\mathbf{N}_{\mathrm{u}}\mathrm{d}\Omega + (\mathrm{i}\omega)^2\int_{\Omega_{\mathrm{p}}} \rho\hat{\lambda}_x\hat{\lambda}_y\hat{\lambda}_z\mathbf{N}_{\mathrm{u}}^{\mathrm{T}}\mathbf{N}_{\mathrm{u}}\mathrm{d}\Omega \tag{10}$$

$$\hat{\mathbf{S}}_{\mathrm{u}\sigma} = \int_{\Omega_{\mathrm{p}}} (\hat{\lambda}_y\hat{\lambda}_z\mathbf{L}_x\mathbf{N}_{\mathrm{u}} + \hat{\lambda}_x\hat{\lambda}_z\mathbf{L}_y\mathbf{N}_{\mathrm{u}} + \hat{\lambda}_x\hat{\lambda}_y\mathbf{L}_z\mathbf{N}_{\mathrm{u}})^{\mathrm{T}}\mathbf{N}_{\sigma}\mathrm{d}\Omega \tag{11}$$

$$\hat{\mathbf{S}}_{\sigma\sigma} = -\int_{\Omega_{\mathrm{p}}} \hat{\lambda}_x\hat{\lambda}_y\hat{\lambda}_z\mathbf{N}_{\sigma}^{\mathrm{T}}\mathbf{D}\mathbf{N}_{\sigma}\mathrm{d}\Omega \tag{12}$$

The system of Eqs. (9) is factorized into a rational form. In order to improve the conditioning of the system and preserve its symmetry, auxiliary stress variables $\underline{\hat{\mathbf{s}}} = (\mathrm{i}\omega\beta)^{-1}\underline{\hat{\boldsymbol{\sigma}}}$ are introduced and the last row of the system is multiplied by $\mathrm{i}\omega\beta$ where the scaling factor β depends on the stiffness and inertial parameters of the FE-PML model:

$$\begin{bmatrix} \hat{\mathbf{S}}_{\mathrm{uu}} & \mathrm{i}\omega\beta\hat{\mathbf{S}}_{\mathrm{u}\sigma} \\ \mathrm{i}\omega\beta\hat{\mathbf{S}}_{\mathrm{u}\sigma}^{\mathrm{T}} & (\mathrm{i}\omega)^2\beta^2\hat{\mathbf{S}}_{\sigma\sigma} \end{bmatrix} \begin{Bmatrix} \underline{\hat{\mathbf{u}}} \\ \underline{\hat{\mathbf{s}}} \end{Bmatrix} = \begin{Bmatrix} \underline{\hat{\mathbf{p}}} \\ \mathbf{0} \end{Bmatrix} \tag{13}$$

The polynomial products $\mathrm{i}\omega\hat{\lambda}_y\hat{\lambda}_z$, $\mathrm{i}\omega\hat{\lambda}_x\hat{\lambda}_z$, $\mathrm{i}\omega\hat{\lambda}_x\hat{\lambda}_y$ and $(\mathrm{i}\omega)^2\hat{\lambda}_x\hat{\lambda}_y\hat{\lambda}_z$ that now appear in Eq. (13) can be written as:

$$\mathrm{i}\omega\hat{\lambda}_y\hat{\lambda}_z = (\mathrm{i}\omega)^{-1}\alpha_{1y}\alpha_{1z} + \alpha_{0y}\alpha_{1z} + \alpha_{1y}\alpha_{0z} + \mathrm{i}\omega\alpha_{0y}\alpha_{0z} = (\mathrm{i}\omega)^{-1}d_{-1} + d_0 + \mathrm{i}\omega d_1 \tag{14}$$

$$\mathrm{i}\omega\hat{\lambda}_x\hat{\lambda}_z = (\mathrm{i}\omega)^{-1}\alpha_{1x}\alpha_{1z} + \alpha_{0x}\alpha_{1z} + \alpha_{1x}\alpha_{0z} + \mathrm{i}\omega\alpha_{0x}\alpha_{0z} = (\mathrm{i}\omega)^{-1}f_{-1} + f_0 + \mathrm{i}\omega f_1 \tag{15}$$