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An orthogonal Hilbert-Huang transform and its application in the spectral representation of earthquake accelerograms

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ABSTRACT

This paper first discusses the limitation that the intrinsic mode functions (IMFs) decomposed by the empirical mode decomposition (EMD) in Hilbert-Huang transform (HHT) are not orthogonal. As an improvement to the HHT method, three orthogonal techniques (the forward, backward and arbitrary sequence orthogonalization algorithms) based on the Gram-Schmidt method are then proposed to obtain the completely orthogonal IMFs. According to the orthogonal index and the energy index, the effectiveness of the proposed technique and algorithms is validated through a synthetic signal generated by the combination of three sinusoidal waves with different frequencies and the El Centro (1940, N-S) earthquake accelerogram. By taking the El Centro (1940, N-S) earthquake accelerogram as an example, the problem that whether the orthogonal IMFs satisfy the requirements of IMF is discussed, then the backward and the arbitrary sequence orthogonalization algorithms are recommended. Three historic earthquake accelerograms are analyzed by using the recommended orthogonalization algorithms combined with the Hilbert spectral analysis. The results show that the orthogonal Hilbert spectrum and the orthogonal Hilbert marginal spectrum can produce more faithful representation of earthquake accelerograms than the Hilbert spectrum and the Hilbert marginal spectrum, and they can be used to quantitatively characterize the energy distribution of earthquake accelerograms at different frequency regions.

1. Introduction

Signal analysis is always the significant and indispensable part in earthquake engineering as well as in many other engineering applications. The characteristics of the earthquake itself and the structures under the action of earthquake can be obtained by analyzing the earthquake ground motion recordings and the seismic responses of structures. The traditional Fourier spectral analysis method, which uses the sine and cosine functions as *a priori* defined bases, has been dominated in the field of earthquake engineering due to its simplicity and high efficiency. However, the Fourier spectral analysis method is theoretically restricted to the linear and stationary signals and cannot fulfil the requirements of earthquake engineering, since the recorded earthquake ground motions and seismic responses of structures are often nonlinear and nonstationary [1,2].

In addition to the Fourier spectral analysis method, many time-frequency analysis methods such as the short-time Fourier transform, Gabor transform, Wigner-Ville distribution and Wavelet transform, have been designed for signal processing [3–7]. However, these methods are all Fourier-based or of Fourier type that transform the time history signal into time-frequency representation through the convolutional computation with respect to an *a priori* selected basis. Therefore, they are suitable for the linear and stationary signals, but are deficient and limited for the nonlinear and nonstationary signals [3–7]. In order to accommodate the nonlinear and nonstationary signals, a new method, Hilbert-Huang transform (HHT), developed by Huang et al. [8,9], has been proposed and widely applied in many engineering fields such as earthquake engineering [2,10–18], geophysics [19–21], system identification [22,23], damage detection [24–27] and structural health monitoring [28] etc. since its introduction. These applications have shown that the HHT is a promising and powerful tool for analyzing the nonlinear and nonstationary signals.

In the applications of the HHT in earthquake engineering, Huang et al. [2] proposed the HHT-based spectral analysis approach, and compared it with the Fourier analysis, wavelet transform, and response spectrum analysis on the Chi-Chi earthquake records recorded at station TCU129, Chi-Chi, Taiwan on 21 September 1999. The results show that the HHT-based spectral analysis can give the most detailed information in the time-frequency-energy representation and the low-frequency energy in the earthquake signal that may be missed by all other

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methods. Huang et al. considered that it is the only spectral analysis method applicable to the nonlinear and nonstationary data. Loh et al. [10,11] applied the HHT-based spectral analysis approach to identify near-fault ground-motion characteristics and structural responses. They concluded that the HHT is a powerful tool for the analysis of frequency-time domain signals and it can detect the time-varying system's natural frequency and damping ratio through the seismic responses of structures. Zhang et al. [12–14] investigated the rational of HHT for analyzing dynamic and earthquake motion recordings in the studies of seismology and engineering. The results show that the HHT is suitable for analyzing nonstationary dynamic and earthquake motion recordings, which is better than the conventional Fourier spectral analysis technique in extracting some features of recordings. Spanos et al. [15] investigated the time-frequency representation of earthquake acclerograms and inelastic structural responses using the adaptive chirplet decomposition combined with the Wigner-Ville transform and the HHT. The analysis for four historic earthquake acclerograms shows that the HHT method can capture the temporal evolution of the frequency content. Dong et al. [16] proposed an improved HHT method and illustrated that it can produce more physically meaningful and readable Hilbert spectrum than the original HHT method, short-time Fourier transform and Wavelet transform through the analysis on the El Centro (1940, N-S) earthquake ground motion recordings. In addition, the HHT method has also been applied to simulate the nonstationary earthquake recordings [17,18]. These applications in earthquake engineering show that HHT can give much finer time-frequency-energy representation and extract more valuable information than the traditional signal analysis methods.

The HHT builds on empirical mode decomposition (EMD) and Hilbert spectral analysis (HSA). It can decompose any complicated dataset via EMD into a finite, often small number of intrinsic mode functions (IMFs) that admit a well-behaved Hilbert transform. Based on the constructed analytical signals of each IMFs by using Hilbert transform, the time-dependent amplitude, the instantaneous frequency of dataset and then the time-frequency distribution of the amplitude are obtained. The EMD is fitter than the Fourier transform and the Fourier-based transform for analyzing the nonstationary data because it decomposes the signal based on the time scale of the signal itself with adaptive nature [8,9].

In the past decade, considerable research has been reported on the algorithm of EMD because it is only an empirical algorithm and has not been proved by strictly mathematical theory. Many approaches and solutions are also proposed to resolve some drawbacks and problems in the algorithm of EMD such as mode mixing, end effects, sifting stopping criterion etc [29–31]. However, limited work has been reported on the problem of the orthogonality of IMFs [32,33]. To address this problem, a new technique based on the Gram-Schmidt orthogonalization method is proposed to improve the orthogonality of IMFs and to obtain the completely orthogonal intrinsic mode functions in this study. Then, three orthogonalization algorithms are suggested and compared. The effectiveness of the proposed technique and algorithms is investigated through a synthetic signal and the El Centro (1940, N-S) earthquake accelerogram using the orthogonal index and the energy index. Furthermore, the recommended orthogonalization algorithms are discussed according to the degree of conformity to the requirements of IMF for the orthogonal IMF components by taking the El Centro (1940, N-S) earthquake accelerogram as an example. The orthogonal Hilbert spectrum and the orthogonal Hilbert marginal spectrum are compared with the original Hilbert spectrum and the original Hilbert marginal spectrum for three historic earthquake accelerograms.

The paper is organized as follows. Section 2 presents the theoretical background of the HHT method and the orthogonal index and the energy index representing the orthogonality of IMFs. Also, the limitation that the IMFs decomposed by EMD are not orthogonal is discussed. Section 3 presents an orthogonal technique based on the Gram-Schmidt method to obtain the completely orthogonal IMFs. Numerical examples to verify the effectiveness the proposed orthogonalization technique are given in Section 4. Section 5 presents the application of the proposed orthogonal Hilbert spectral analysis method in the earthquake accelerograms. Finally, some conclusions are drawn in Section 6.

2. The Hilbert-Huang transform

2.1. Empirical mode decomposition

The HHT is the combination of EMD and HSA. The EMD method is designated to decompose any nonlinear and nonstationary signals into a finite set of oscillatory modes by using a procedure called sifting process. The oscillatory modes are defined as the IMFs that the following two necessary conditions are required, (1) within the data range, the number of extrema and the number of zero crossings of an IMF must either equal or differ at most by one; and (2) at any point, the envelope defined by the local maxima and the envelope defined by the local minima are symmetric with respect to the mean.

The sifting process used in EMD for extracting IMF is briefly introduced. Suppose $X(t)$ is the signal to be decomposed. First, identify all the local extrema and connect all the local maxima and minima by a cubic spline to produce the upper and lower envelope. The mean of these two envelopes is designated as $m_1(t)$, and the difference between the data $X(t)$ and $m_1(t)$ is $h_1(t) = X(t) - m_1(t)$. h_1 will be an IMF, if it satisfies the above two necessary conditions. Yet, in general h_1 doesn't satisfy these conditions. Then, in the subsequent sifting process, h_1 is treated as a new data. After repeated sifting up to k times, when h_{1k}, which is given by $h_{1k}(t) = h_{1(k-1)}(t) - m_{1k}(t)$ satisfies the above two necessary conditions, h_{1k} is designated as the first IMF $c_1(t)$ from the data, i.e. $c_1(t) = h_{1k}(t)$. Next, one removes $c_1(t)$ from the rest of the data to obtain the residue $r_1(t)$ which is given by $r_1(t) = X(t) - c_1(t)$. $r_1(t)$ is treated as the new data and subjected to the same sifting process as described above, giving the second IMF $c_2(t)$. The sifting process will be terminated when either the component $c_n(t)$ or the residue $r_n(t)$ is less than a predetermined value or the residue $r_n(t)$ becomes a monotonic function. Thus, the original signal $X(t)$ is finally expressed as the sum of n IMF components plus the final residue,

$$X(t) = \sum_{j=1}^{n} c_j(t) + r_n(t) \tag{1}$$

where $c_j(t)$ $(j = 1, 2, ..., n)$ is the jth IMF component; and $r_n(t)$ is the final residue. The obtained IMF components are sequenced from high frequency to low frequency and are nearly orthogonal to each other.

2.2. Hilbert spectral analysis

For an arbitrary function $x(t)$, its Hilbert transform $y(t)$ is given by the convolution of the function $x(t)$ with $1/t$, namely

$$y(t) = \frac{P}{\pi}\left(\int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau\right) \tag{2}$$

where P is the Cauchy principle value. Coupling the $x(t)$ and $y(t)$, the analytic signal is defined as

$$z(t) = x(t) + \mathrm{i}y(t) = a(t)e^{\mathrm{i}\theta(t)} \tag{3}$$

where

$$a(t) = \sqrt{x^2(t) + y^2(t)},\ \ \theta(t) = \arctan(y(t)/x(t)) \tag{4}$$

in which $a(t)$ is the instantaneous amplitude of $x(t)$, and $\theta(t)$ is the instantaneous phase of $x(t)$. The instantaneous frequency is defined as the derivative of the phase $\theta(t)$ given by

$$\omega(t) = \frac{d\theta(t)}{dt} \tag{5}$$

The definition of instantaneous frequency is only meaningful for