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Static and seismic earth pressure coefficients for vertical walls with horizontal backfill

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ABSTRACT

The seismic earth pressure problem is considered for the special case of a vertical wall with horizontal backfill. Using upper and lower bound finite element limit analysis, earth pressure coefficients are derived for a range of seismic coefficients and soil-wall interface friction angles. The coefficients presented have a verifiable error of at most ± 1%. Finally, the earth pressure coefficients are applied in a limit equilibrium framework to the design of various embedded retaining structures. For the resulting designs, factors of safety are computed using upper and lower bound strength reduction finite element analysis and it is concluded that the limit equilibrium approach, using the new earth pressure coefficients, is fairly accurate, albeit slightly unconservative.

1. Introduction

The design of retaining walls is often carried out using limit equilibrium approaches where the earth pressures acting on the wall are derived from the classic earth pressure problem first considered by Coulomb [3]. A distinction is made between active pressures tending to destabilize the wall and passive pressures resisting the movement of the wall. Each pressure is characterized by an earth pressure coefficient which depends on the internal soil friction angle, the soil-wall interface friction angle, the geometry of the wall and the slope of the backfill.

In the static case, a very large number of solutions and associated expressions for the active and passive earth pressure coefficients have been proposed. Besides the original contribution of Coulomb, classic works include those of Rankine [20], Caquot [1] and Sokolovski [22]. More recent contributions can be found in [6,2,12,23,5,19]. Furthermore, many codes of practice, e.g. Eurocode 7 [4], include tabulated or closed-form expressions for the earth pressure coefficients. Despite this abundance of solutions, there are still no exact solutions (except for the case of a smooth wall) and there is little agreement regarding the accuracy of the available approximate solutions.

A common approach to the design of retaining walls under seismically induced loading is the so-called pseudo-static approach. The standard static problem is here augmented by body forces representing the relevant inertial forces. The magnitude of these are given as fractions of the vertical body force due to self-weight.

By employing seismic earth pressure coefficients, which now depend also on the magnitude of the equivalent internal forces relative to the static ones, this approach extends readily to the common limit equilibrium approach. Starting from the work of Mononobe and Okabe (see e.g. [11]) and following the development for the static earth pressure problem, a large number of expressions for the analogous seismic earth pressure coefficients have been proposed (see e.g. [17,21,18], and references therein). As in the static case, there appears to be little consensus regarding the accuracy of the proposed seismic earth pressure coefficients.

The aim of the present note is to present a set of accurate active and passive seismic earth pressure coefficients. These are computed using finite element limit analysis (FELA), thus allowing for the determination of rigorous upper and lower bounds and, in turn, for rigorous estimates of the accuracy of the solutions established. A subset of the most general earth pressure problem is considered, namely that involving a vertical wall supporting a purely frictional backfill with zero inclination to the horizontal. Only horizontal seismic loading is considered.

Secondly, the standard limit equilibrium approach, employing the new earth pressure coefficients, is applied to a number of stability problems involving embedded retaining walls. The solutions, which are compared to rigorous upper and lower bound strength reduction finite element limit analysis (SR-FELA) are shown to be fairly accurate, albeit slightly unconservative.

2. Earth pressure coefficients

The classic earth pressure problem is shown in Fig. 1. The wall is rigid and is constrained against vertical displacement and rotation. The soil is purely frictional and characterized by the internal soil friction angle ϕ while the interface between the soil and the wall is

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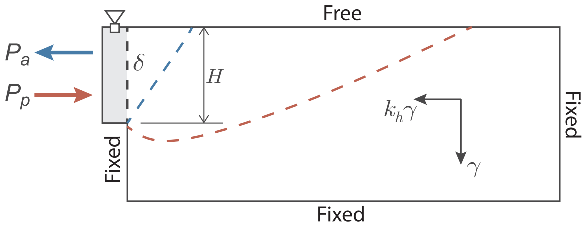

Fig. 1. Earth pressure problem.

Table 1
Active earth pressure coefficients (max error = ±0.5%).

	δ/ϕ				
$\phi(^\circ)$	0	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	1
10	0.704	0.674	0.662	0.652	0.637
15	0.589	0.555	0.541	0.530	0.513
20	0.490	0.456	0.442	0.431	0.413
25	0.406	0.374	0.361	0.350	0.332
30	0.333	0.305	0.293	0.283	0.266
35	0.271	0.246	0.236	0.227	0.212
40	0.217	0.197	0.188	0.181	0.167
45	0.172	0.155	0.148	0.142	0.130

Table 2
Passive earth pressure coefficients (max error = ±0.5%).

	δ/ϕ				
$\phi(^\circ)$	0	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	1
10	1.42	1.51	1.55	1.58	1.63
15	1.70	1.88	1.96	2.03	2.14
20	2.04	2.37	2.52	2.66	2.86
25	2.46	3.03	3.31	3.57	3.95
30	3.00	3.96	4.46	4.94	5.68
35	3.69	5.31	6.23	7.15	8.58
40	4.60	7.38	9.09	10.93	13.9
45	5.83	10.72	14.13	18.03	24.72

Table 3
Strutted sheet pile wall designs using limit equilibrium and factor of safeties using strength reduction finite element limit analysis. The maximum error in the SR-FELA results is ± 0.005.

	Limit equilibrium			SR-FELA
k_h	D (m)	P (kN/m)	M (kNm/m)	FS
0.00	2.40	136	390	0.984
0.05	2.60	152	442	0.983
0.10	2.86	172	511	0.982
0.15	3.16	195	593	0.980
0.20	3.53	225	701	0.980
0.25	3.98	261	838	0.978
0.30	4.56	308	1026	0.977

characterized by the friction angle δ. The soil mass is subjected to a downward force γ stemming from self-weight and a horizontal force $k_h\gamma$ representing inertial forces with k_h being denoted the horizontal seismic coefficient. The wall is subjected to forces P_a and P_p in the active and passive cases respectively as shown in the figure and the soil domain is large enough to contain the failure mechanisms as indicated.

In the static case ($k_h = 0$), the active and passive earth pressures coefficients, K_a and K_p respectively, are defined via the active and passive thrusts as

Table 4
Cantilever sheet pile wall designs using limit equilibrium and factor of safeties using strength reduction finite element limit analysis. The maximum error in the SR-FELA results is ± 0.005.

	Limit equilibrium			SR-FELA
k_h	d (m)	D (m)	M (kNm/m)	FS
0.00	3.58	3.97	317	0.978
0.05	3.84	4.24	362	0.978
0.10	4.17	4.58	421	0.982
0.15	4.54	4.97	493	0.984
0.20	5.00	5.45	589	0.986
0.25	5.56	6.01	712	0.986
0.30	6.27	6.75	885	0.988

$$P_a = \frac{1}{2}\gamma H^2 K_a, \quad P_p = \frac{1}{2}\gamma H^2 K_p \tag{1}$$

In the seismic case, a distinction is made between the direction of seismic acceleration relative to the movement of the wall. For cases where the wall movement is in the direction of acceleration, the active earth pressure coefficient is denoted by K_a^+ and the passive coefficient by K_p^-. Conversely, for cases where the wall movement is opposite the direction of acceleration, the active and passive earth pressure coefficients are denoted by K_a^- and K_p^+ respectively. With this nomenclature, we always have $K_a^- \le K_a \le K_a^+$ and $K_p^- \le K_p \le K_p^+$. For $k_h > 0$, the setup in Fig. 1 correspond to the '+' cases (both coefficients are greater than in the static case) while $k_h < 0$ corresponds to the '−' cases (both coefficients are less than in the static case).

All results presented in this note have been obtained by means of finite element limit analysis (see e.g. [13,14,7,8,15,16]) using the program OptumG2 [10]. This technique allows for the computation of rigorous upper and lower bounds on the collapse load of structures of rigid-plastic material. Moreover, with rigorous upper and lower bounds at hand, it is possible to compute a rigorous estimate of the worst case error. For a given problem, let the lower bound collapse load be denoted by L and the upper bound collapse load by U. An estimate of true solution then follows simply as the mean value of the upper and lower bounds:

$$M = \frac{L + U}{2} \tag{2}$$

Furthermore, let the exact solution be denoted by E. Since $L \le E \le U$, we have

$$M(1 - \varepsilon) \le E \le M(1 + \varepsilon) \tag{3}$$

where

$$\varepsilon = \frac{U - L}{U + L} \tag{4}$$

is the relative worst case error. For given upper and lower bound meshes of roughly the same quality, experience shows that the actual error in the mean values usually is much less than this estimate. That is, the lower bound tends to be underestimated by an amount similar to which the upper bound is overestimated, leading to the mean value being an excellent estimate of the exact solution.

With the above considerations, static active and passive earth pressure coefficients with a maximum error of 0.5% are presented in Table 1 for a range of internal and soil-wall friction angles Table 2. To the Author's knowledge, these coefficients have not previously been published . The closest agreement with any published coefficients appear to be with those shown graphically in Figure C.2.1 of Eurocode 7 [4] .

The seismic coefficients are shown in Tables 5–8. These come with a maximum error of ± 1%. It is noted that the feasible seismic coefficient is limited by $|k_h| \le \tan\phi$.