Soil Dynamics and Earthquake Engineering 85 (2016) 191–201

Contents lists available at ScienceDirect

Soil Dynamics and Earthquake Engineering

journal homepage: www.elsevier.com/locate/soildyn

A refined analytical model for earthquake-induced sloshing in half–full deformable horizontal cylindrical liquid containers

Spyros A. Karamanos*, Angeliki Kouka

University of Thessaly, Volos 38334, Greece

ARTICLE INFO

Article history:
Received 19 December 2015
Received in revised form
29 February 2016
Accepted 3 March 2016
Available online 17 April 2016

Keywords:
Seismic response
Liquid container excitation
Horizontal-cylindrical container
Fluid-structure interaction
Sloshing

ABSTRACT

The coupled response of elastic deformable liquid containers of horizontal-cylindrical shape under external seismic excitation is examined, through an analytical methodology, assuming inviscid-incompressible fluid and irrotational-flow conditions. In particular, the case of a half–full horizontal-cylindrical deformable container is examined, considering an analytical series-type solution for the velocity potential function that describes the liquid motion under external excitation. This mathematical analysis extends the solution methodology presented in previous publications of the senior author, taking into account full coupling between sloshing and wall deformation in a rigorous manner, where wall deformation is considered through a sinusoidal assumed-shape function. In the mathematical formulation, the velocity potential is decomposed into three parts: (a) a first part, which represents liquid motion that follows the external excitation, (b) a "convective part", representing liquid motion associated with free surface elevation (sloshing), and (c) a third part caused by the wall deformation. Using an elegant mathematical manipulation, the coupled transient overall response of the liquid-container system is obtained in an efficient manner. Numerical results are presented in terms of the principal natural frequencies of the coupled system, as well as the system response under strong seismic input, and emphasize on the effects of container aspect ratio on the dynamic behavior of the system. The mathematical formulation for the case of long cylinders results in a simplified model, identical to the simplified "physical model" presented in a previous publication.

1. Introduction

The presence of a free surface in partially filled liquid containers allows for fluid motions relative to the container, associated with free-surface elevation. This phenomenon, referred to as "liquid sloshing", is generally caused by external tank excitation, and may have a significant influence on the response of the container. Assuming ideal fluid, the fluid flow is described through a velocity potential function satisfying the Laplace equation within the fluid, the kinematic condition on the tank wall, and the kinematic and dynamic free-surface conditions. Furthermore, considering small amplitude conditions, a linearized condition on the free surface of liquid is obtained. In the absence of external excitation, sloshing can be regarded as an eigenvalue problem, which represents the oscillations of the free surface of an ideal liquid inside a stationary container. The eigen-problem solution provides the natural frequencies of fluid oscillation (sloshing frequencies) and the corresponding sloshing modes, and depends strongly on the shape of the container. In the case of externally excited container, sloshing becomes a transient problem. The total liquid motion can be decomposed in two parts, first part which represents liquid motion that follows the external excitation, and a second part associated with sloshing, which expresses fluid motion with respect to the container. The solution of the transient problem provides the hydrodynamic pressures and forces on the container's wall.

In non-deformable liquid containers of rectangular and vertical-cylindrical shape, the sloshing problem can be solved analytically, using separation of variables, and the corresponding sloshing modes are mutually orthogonal and uncoupled. For other geometries (e.g. horizontal cylinders or spheres) exact analytical solutions may not be available, and the use of numerical or semi-numerical methods becomes necessary. Sloshing frequencies in non-deformable circular cylinders (canals) as well as the corresponding transient problem of externally-induced sloshing has been studied numerically in an early work by [1], using space transformations to map the initial circular region to a more convenient plane region. The flow field was described by a set of integral equations, which was solved using a Galerkin-type solution. Further contributions on the calculation of sloshing

* Corresponding author.
E-mail address: skara@mie.uth.gr (S.A. Karamanos).

http://dx.doi.org/10.1016/j.soildyn.2016.03.004

frequencies in horizontal cylindrical containers filled up to an arbitrary height have been reported by Moiseev and Petrov [2], and later by [3–7] using numerical or semi-numerical methods.

Recently, the analysis of horizontal cylindrical liquid containers under external excitation has received quite some attention, mainly because of its application in dynamics and stability of moving vehicles containing a liquid with a free surface. Faltinsen and Timokha [8,9] presented a multimodal method for two-dimensional forced liquid sloshing in a circular container, which employs an expansion in terms of the natural sloshing modes. The multimodal method has also been used by [10,11] to analyse sloshing in a moving horizontal-cylindrical container of both circular and general cross-sectional shape. Furthermore, several semi-numerical and numerical works have been motivated by the response of horizontal cylindrical pressure vessels under seismic loading [12–14], whereas the reader is referred to the recent paper by Malhotra [15] for an important application of this topic in the seismic design of a large-scale horizontal-cylindrical container in the International Thermonuclear Experimental Reactor (ITER), in France.

The particular case of a half-full horizontal cylindrical container offers the possibility for developing an elegant analytical formulation, which can be used as benchmark for verifying numerical methodologies. Evans and Linton [16] developed a series-type analytical solution of the eigenvalue sloshing problem of a half-full two-dimensional liquid container, expanding the velocity potential in a series of non-orthogonal bounded harmonic spatial functions. This series solution has been extended by the authors [17,18] for the calculation of hydrodynamic pressures and forces in half-full cylinders under transverse and longitudinal external excitation respectively, expanding the velocity potential in bounded series in terms of arbitrary time functions and their associated non-orthogonal spatial functions, resulting in a system of ordinary linear differential equations. In the case of transverse excitation, a simplified three-dimensional model was also presented by Papaspyrou et al. [18] and further developed in Karamanos et al. [19], extending the series solution to account for wall deformation and calculating the coupled response of the liquid-structure system. More recently, Hasheminejad and Aghabeigi [20,21] analyzed sloshing in half-full horizontal cylindrical containers of elliptical cross-section and in a subsequent publication they extended their formulation to examine the effects of vertical or horizontal side baffles on sloshing response [22].

The present work, motivated by the seismic analysis of horizontal-cylindrical pressure vessels [19], is aimed at developing a rigorous mathematical model to calculate sloshing effects in deformable half-full horizontal cylindrical containers under external excitation in the transverse direction, extending and refining the work presented in Papaspyrou et al. [18]. The coupled liquid-structure response is tackled through an analytical methodology, considering the influence of container wall motion on liquid sloshing, through an appropriate assumed shape function to account for vessel deformation. It should be noted that for the case of deformable containers, the “sloshing” or “convective” motion has been customarily considered neglecting wall deformation effects. Such an approach has been used extensively in the seismic analysis of vertical cylindrical liquid storage tanks, where the container's deformation was taken into account through either simple assumed-shape functions [23,24], or more elaborate shell deformation models [25–27]. In the present work, the case of half-full horizontal cylinders under external transverse excitation is examined using a mathematical formulation that allows for full coupling between liquid motion and wall deformation through an explicit and rigorous manner. A similar approach has been followed by Fischer and Rammerstorfer [28] for the case of upright cylindrical liquid containers. The formulation decomposes the motion in three parts: a first part that follows the external source, a second part due to container deformation and a third part associated with sloshing of the liquid free surface. In the present study, following the terminology widely adopted in the literature for earthquake response of liquid storage tanks (e.g. [23,24]; and [28]), these three parts are referred to as “impulsive” motion, “deformation” motion and “convective” motion respectively.

A truncated solution is also developed considering only the first two terms of the series in the transverse direction, which yields an elegant solution of good accuracy and enables the parametric study. The particular case of harmonic excitation is also examined, which results in a system of algebraic equations. Comparison of the present rigorous approach with the more simplified approach proposed by Papaspyrou et al. [18] is conducted towards better understanding the effects of container wall deformation on the overall dynamic response of the half-full horizontal cylinder. The numerical results are presented in terms of the frequencies of the coupled system with respect to the aspect ratio of the container, as well as the response the liquid-container system under external excitation from a severe seismic event for different values of the aspect ratio.

2. Formulation of the coupled problem

The fluid is contained in a half-full horizontal cylindrical vessel of radius R, with the y-axis of the coordinate system x, y, z, where axis y points vertically downwards (Fig. 1). The geometry is described in terms of the cylindrical coordinates r, θ, z.

2.1. Vessel deformation

The container undergoes an arbitrary motion of its supports in the direction of the x axis with displacement $X(t)$. The vessel is assumed flexible (deformable) in the form of a beam-type deformation where the cross-section remains circular (those vessels are rather thick to resist high internal pressure). However, relatively long horizontal cylindrical vessels ($L/R \geq 10$), quite common in petrochemical industries and refineries, exhibit a beam-type deformation, which may affect the overall response under transverse excitation. Thus, neglecting local (shell-type) modes, while the cylinder cross-section remains circular (undeformed) due to its significant thickness, the motion of the cylindrical container is directly determined by the motion of the cylinder axis, which is decomposed in two parts (Fig. 2), the motion of the supports $X(t)$, independent of z coordinate, and the motion due to the

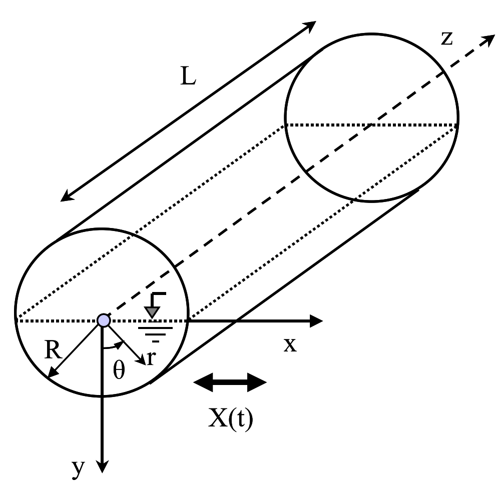

Fig. 1. Schematic representation of the problem: half-full horizontal cylindrical container and coordinate system.