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Dynamic response of axisymmetric transversely isotropic viscoelastic continuously nonhomogeneous half-space

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ABSTRACT

An analytical derivation is presented for dynamic response of axisymmetric transversely isotropic linear viscoelastic continuously nonhomogeneous half-space subjected to vertical either point or circular patch load. The material coefficients are considered to vary in terms of depth as bounded exponentially functions, and the mass density is assumed to be constant. Hankel integral transforms accompanied with Frobenius series method are applied to solve the boundary value problem. The unknown constants are determined by satisfying boundary conditions and regularity conditions at infinity, after which the displacement and stress fields are specified in Hankel space. The inverse Hankel integral transforms are utilized to specify the displacements and stresses in real domain. It is shown that inhomogeneity parameters affect the dynamic response of the half-space considerably, especially at the vicinity of the free-surface. Moreover, viscoelastic behavior of the half-space is parametrically studied assuming several damping ratios, and it is seen that it can considerably change the dynamic responses especially for long horizontal distances from the excitation point.

1. Introduction

Elastic moduli of the soil usually varies with depth mainly due to dependence to effective confining pressure and consequently, wave velocity varies with depth as demonstrated by resonant column experiment and also in-situ measurements such as cross-hole tests for isotropic materials [1]. On the other hand, different horizontal and vertical pressures of the soil medium and also its sedimentation process leads to transversely isotropic behavior of the soil. Furthermore, when subjected to time-harmonic load, the soil medium can dissipate some parts of energy, which can be considered assuming material viscoelastic behavior [2]. In the other words, a realistic soil half-space has three main characteristics as inhomogeneity, transversely isotropy, and viscoelasticity, each of which can affect wave propagation in the media and dynamic response of the foundations resting on the soil and thus can affect the results of realistic soil-structure-interaction analysis. Because of these, the geotechnical engineers are interested in studying wave propagation in nonhomogeneous anisotropic viscoelastic domains. On the other hand, mathematically, investigation of the boundary value problem of time-harmonic wave propagation in this kind of material is more difficult, and thus the mathematicians are also interested in the subject.

A summary list of several studies done in this area along with main assumptions and solutions has been presented by Wang et al. [3]. There exists a few researches studying dynamic response of continuously inhomogeneous transversely isotropic half-space with some simplified assumptions; however, to the best knowledge of the authors, there is no research in the literature investigating wave propagation in inhomogeneous viscoelastic transversely isotropic half-space assuming a bounded variation of elastic moduli with depth.

One of the earliest works in the continuously inhomogeneous isotropic media was done by Stoneley [4], who studied transmission of Rayleigh waves in nonhomogeneous half-space assuming linear variation of soil shear modulus in an incompressible medium. Nearly at the same time, Pekeris [5] derived Rayleigh wave velocity in nonhomogeneous medium and Richter [6] investigated body wave propagation in inhomogeneous medium. Several studies are done by Gibson and co-researchers. For example, the static displacement response of heterogeneous incompressible isotropic medium has been investigated by Gibson et al. [7] and Gibson and Kalsi [8] assuming a linear variation for the shear modulus. Gazetas [9] derived both the static and dynamic displacements of foundations rested on the top of a continuously inhomogeneous multilayered soil assuming second order variation of shear modulus and thus linear variation of shear wave velocity in terms of depth. Also, Gazetas [10] studied dynamic characteristics of soil deposits such as natural period, mode shapes and amplification function for several variations of shear modulus.

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Nomenclature

A_i $(i=1\sim4)$	Coefficients determined by boundary and regularity conditions
a	Radius of circular patch load
a_n, b_n	Coefficients of different terms of power series
C_{0ij}	Elastic moduli at free surface $z=0$
$C_{\infty ij}$	Elastic moduli at infinite depth
$C_{0ij}/C_{\infty ij}$	inhomogeneity parameter
E, E_0	Young's moduli in the plane of transverse isotropy in general and at free surface $z=0$
E', E'_0	Young's moduli in the direction normal to the plane of transverse isotropy in general and at free surface $z=0$
G, G_0	shear modulus in the plane normal to the axis of symmetry in general and at free surface $z=0$
G', G'_0	shear modulus in planes normal to the plane of transverse isotropy in general and at free surface $z=0$
J_m	Bessel function of the first kind and m^{th} order
L	Unit length for point load case or a (radius) for circular patch load case
$R(r)$	time-harmonic surface force component in z direction
r	radial coordinate
t	time variable
u_i	displacement component in i direction $(i=r,z)$
z	vertical coordinate
β	inhomogeneity parameter
$\delta(r)$	Dirac-delta function
$\varepsilon_{ij}(i,j=r,\theta,z)$	strain components
η	transformed depth variable
θ	angular coordinate
ρ	material density
$\sigma_{ij}(i,j=r,\theta,z)$	stress tensor
υ	Poisson's ratio, $\upsilon=-\varepsilon_{\theta\theta}/\varepsilon_{rr}$ when subjected to the stress σ_{rr} or $\upsilon=-\varepsilon_{rr}/\varepsilon_{\theta\theta}$ when subjected to the stress $\sigma_{\theta\theta}$
υ'	Poisson's ratio, $\upsilon'=-\varepsilon_{rr}/\varepsilon_{zz}=-\varepsilon_{\theta\theta}/\varepsilon_{zz}$ when subjected to the stress σ_{zz}
ω_0	non-dimensional frequency
ω	angular frequency
ξ	Hankel's integral transform parameter
ζ	Material damping ratio

Selvadurai et al. [11] introduced a bounded variation of shear modulus as $G(z)=G_\infty+(G_0-G_\infty)e^{-\beta z}$ and investigated Reissner–Sagoci problem for non-homogeneous isotropic solid in the static case. This distribution function can be matched with realistic soil characteristics in many cases and was used by some researchers after introducing it. Vrettos has investigated different aspects of wave propagation in vertically continuously inhomogeneous isotropic half-space in several studies as [12–23]. Mostly, he used the bounded variation of shear modulus introduced by Selvadurai et al. [11] in his researches. Chapman [24] studied seismic wave propagation in isotropic homogeneous, anisotropic homogenous and isotropic heterogeneous media, mainly using ray theory. Researches in the topic interested in this paper show that the wave propagation and dynamic response of continuously non-homogeneous isotropic half-space contain the following main differences compared with homogeneous one:

a) The wave spread in a curved path in continuously non-homogeneous media because of continuous change in stiffness and wave velocity with depth [24].
b) The displacement response of the continuously non-homogeneous half-space changes with different patterns along vertical axis and free surface [16,25].
c) In continuously nonhomogeneous half-spaces several modes of dispersive surface waves are produced, while in homogeneous isotropic half-space, only one non-dispersive surface wave is produced denoted as Rayleigh surface wave [15].
d) Dispersive SH surface waves are produced in continuously nonhomogeneous half-space [14], while in homogeneous not layered half space no SH surface wave exist.
e) Dispersive torsional surface waves are produced in continuously nonhomogeneous half-space [26].

In addition to studying the static response of anisotropic half-spaces to traction/displacement boundary conditions due to existence of foundations, many researchers have studied wave propagation and dynamic response of foundations in homogeneous transversely isotropic medium. Some of early works are presented by Morse [27], Buchwald [28] and Anderson [29], who studied wave propagation in transversely isotropic media using several simplified assumptions. Waas et al. [30] investigated the dynamic response of transversely isotropic stratified media, which is frequently referred by many later authors. Rajapakse and Wang [31] presented Green's function for transversely isotropic elastic half-space. Another frequently referred study was presented by Eskandari–Ghadi [32], who derived potential functions appropriate for solving the governing equations in transversely isotropic media. Several other studies have been done to deal with the wave propagation and dynamic response in homogeneous transversely isotropic media such as [33–42]. These researches show that the wave propagation and also dynamic response in transversely isotropic media are different from isotropic one depending on degree of anisotropy of the transversely isotropic material.

There are only a few researches investigating the wave propagation or dynamic response of inhomogeneous transversely isotropic media. Recently, Wang et al. have presented displacement and stress response of inhomogeneous transversely isotropic half-space assuming exponentially decreasing distribution of elasticity tensor as $C_{ij}(z)=C^0_{ij}\,e^{-kz}$ in the static case [43,44]. Then, he studied wave propagation in inhomogeneous transversely isotropic media using the same distribution [3] and also using similar distribution for elastic moduli and density as $C_{ij}(z)=C^0_{ij}(a+bz)^c$ and $\rho(z)=\rho^0(a+bz)^c$, respectively, [45]. It should be noted that in these studies, wave velocity does not change in depth because of similar kind of inhomogeneities considered for both elasticity tensor and mass density. Eskandari-Ghadi and Amiri-Hezaveh [46] derived new potential functions and investigated wave propagation in exponentially graded transversely isotropic half-space assuming similar distribution for elastic moduli and density as $C_{ij}(z)=C^0_{ij}\,e^{2\beta z}$ and $\rho(z)=\rho^0\,e^{2\beta z}$.

There exists some researches for the wave propagation in viscoelastic media, such as [47–49]. However, there exist only a few materials investigating dynamic response of foundations resting on or buried in viscoelastic media. Veletsos and Verbič [50] studied vibration of rigid circular foundation on viscoelastic half-space. Several other papers were presented by Luco determining impedance functions of different foundation shapes resting on viscoelastic half-space (see [51,52] as examples). Moreover, Gazetas [53] determined dynamic compliance matrix of rigid strip footing attached on viscoelastic transversely isotropic half-space.

In this paper, the axisymmetric wave propagation in a more realistic continuously nonhomogeneous material is considered, where elastic modulus varies in depth using a bounded distribution within