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Technical Note

Dynamic analysis of a transversely isotropic multilayered half-plane subjected to a moving load

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ABSTRACT

In this paper, the analytical layer-element method is utilized to analyze the plane strain dynamic response of a transversely isotropic multilayered half-plane due to a moving load. We assume that the studied system moves synchronously with the moving load, then the moving load relative to the moving system is considered to be motionless. Therefore, the vertical stress and the vertical displacement under the moving load need not update for the variation of the load position. Based on the governing equations of motion in the moving system, the analytical layer-element solutions for a finite layer and a half-plane in the Fourier transform domain are derived by using the algebraic operations in Ref. [7]. The global matrix of the problem can be obtained by assembling the analytical layer-elements of all layers. The corresponding solution in the frequency domain is further recovered by the inverse Fourier transform. Several examples are given to confirm the accuracy of the proposed method and to illustrate the influence of material properties.

1. Introduction

The dynamic response of the moving load is widely applied in practical engineering like railroads, highways, bridges and runways. Many models and methods of analysis for the problem have been proposed, and Beskou and Theodorakopoulos [1] gave a comprehensive review on the dynamic response of homogenous or layered half-space subjected to a moving load. Apart from the character of layering, soils like as London clay [2] also take on the phenomenon of transverse isotropy. Rajapakse and Wang [3] derived Green's functions for a transversely isotropic elastic half-space subjected to time-harmonic excitations. Shodja and Eskandari [4] studied the axisymmetric time-harmonic response of a transversely isotropic substrate-coating system. Khojasteh et al. [5] obtained the three-dimensional dynamic Green's functions for a transversely isotropic multilayered half-space. Recently, Lin [6] presented a numerical approach to calculate the Green's function for an anisotropic multilayered half-space by employing the Fourier transform and the precise integration method.

From the above literature review, few researches consider a transversely isotropic multilayered half-plane subjected to a moving load. In this paper, the dynamic response of a transversely isotropic multilayered half-plane subjected to a moving load is solved by using the analytical layer-element method [7]. For the velocity of the moving load in practical engineering is generally in the subsonic range, only the subsonic case is studied here. Based on the governing equations of motion in the moving system, the global stiffness matrix in the transformed domain for a transversely isotropic multilayered half-plane due to a moving load is derived by using the algebraic operations in Ref. [7] as reference. The corresponding solution in the frequency domain is further recovered by the inverse Fourier transform. At last, several numerical examples are performed to confirm the accuracy of present method and to discuss the influence of material properties.

2. The governing equations of motion in the moving coordinate system

As shown in Fig. 1, let X and Z be Cartesian coordinates fixed in the medium occupying the half-space $Z \geq 0$. Consider a load of intensity q and width $2a$ acting along the line $Z=0$ moves in the positive X direction with a constant speed v for a long time.

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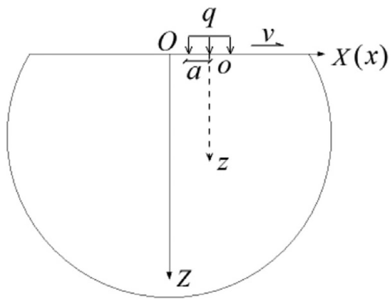

Fig. 1. Steadily moving load on the surface of transversely isotropic half-plane.

In plane-strain conditions, the governing equations of motion in the absence of body forces for a transversely isotropic material are [7]

$$c_{11}\frac{\partial^2 u_X}{\partial X^2}+c_{44}\frac{\partial^2 u_X}{\partial Z^2}+(c_{13}+c_{44})\frac{\partial^2 u_Z}{\partial X\partial Z}=\rho\frac{\partial^2 u_X}{\partial t^2} \tag{1a}$$

$$(c_{13}+c_{44})\frac{\partial^2 u_X}{\partial X\partial Z}+c_{33}\frac{\partial^2 u_Z}{\partial Z^2}+c_{44}\frac{\partial^2 u_Z}{\partial X^2}=\rho\frac{\partial^2 u_Z}{\partial t^2} \tag{1b}$$

where u_X and u_Z represent the displacement components in the X and Z directions, respectively; ρ denotes the mass density; t is the time. In addition, $c_{11}=\lambda n(1-n\mu_{vh}^2)$, $c_{12}=\lambda n(\mu_h+n\mu_{vh}^2)$, $c_{13}=\lambda n\mu_{vh}(1+\mu_h)$, $c_{33}=\lambda(1-\mu_h^2)$, $c_{44}=G_v$, $n=E_h/E_v$ and $\lambda=E_v/[(1+\mu_h)(1-\mu_h-2n\mu_{vh}^2)]$, in which E_h, E_v and G_v represent the horizontal Young's modulus, vertical Young's modulus and shear modulus normal to the half-plane, respectively, and μ_{vh} and μ_h are the Poisson's ratios expressing lateral strain as a result of stress acting parallel and normally to the plane, respectively.

To replace the fixed system (X,Z), now we introduce a moving system (x,z) which moves along with the moving load q at the same time (see Fig. 1). It is defined that u_x and u_z are the displacement components along x and z axis in the moving coordinate system, hence,

$$x=X-vt$$
$$z=Z \tag{2a}$$

$$\frac{\partial u_X}{\partial X}=\frac{\partial u_x}{\partial x},\ \frac{\partial u_X}{\partial Z}=\frac{\partial u_x}{\partial z},\ \frac{\partial u_X}{\partial t}=\frac{\partial u_x}{\partial t}-v\frac{\partial u_x}{\partial x}$$
$$\frac{\partial u_Z}{\partial X}=\frac{\partial u_z}{\partial x},\ \frac{\partial u_Z}{\partial Z}=\frac{\partial u_z}{\partial z},\ \frac{\partial u_Z}{\partial t}=\frac{\partial u_z}{\partial t}-v\frac{\partial u_z}{\partial x} \tag{2b}$$

$$\frac{\partial^2 u_X}{\partial X^2}=\frac{\partial^2 u_x}{\partial x^2},\ \frac{\partial^2 u_X}{\partial Z^2}=\frac{\partial^2 u_x}{\partial z^2},\ \frac{\partial^2 u_X}{\partial t^2}=\frac{\partial^2 u_x}{\partial t^2}-2v\frac{\partial^2 u_x}{\partial x\partial t}+v^2\frac{\partial^2 u_x}{\partial x^2}$$
$$\frac{\partial^2 u_Z}{\partial X^2}=\frac{\partial^2 u_z}{\partial x^2},\ \frac{\partial^2 u_Z}{\partial Z^2}=\frac{\partial^2 u_z}{\partial z^2},\ \frac{\partial^2 u_Z}{\partial t^2}=\frac{\partial^2 u_z}{\partial t^2}-2v\frac{\partial^2 u_z}{\partial x\partial t}+v^2\frac{\partial^2 u_z}{\partial x^2} \tag{2c}$$

Substituting Eqs. (2) into Eqs. (1) leads to the governing equations of motion for the transient response of an elastic body in the moving coordinate system as follows:

$$c_{11}\frac{\partial^2 u_x}{\partial x^2}+c_{44}\frac{\partial^2 u_x}{\partial z^2}+(c_{13}+c_{44})\frac{\partial^2 u_z}{\partial x\partial z}=\rho\left(\frac{\partial^2 u_x}{\partial t^2}-2v\frac{\partial^2 u_x}{\partial x\partial t}+v^2\frac{\partial^2 u_x}{\partial x^2}\right) \tag{3a}$$

$$(c_{13}+c_{44})\frac{\partial^2 u_x}{\partial x\partial z}+c_{33}\frac{\partial^2 u_z}{\partial z^2}+c_{44}\frac{\partial^2 u_z}{\partial x^2}=\rho\left(\frac{\partial^2 u_z}{\partial t^2}-2v\frac{\partial^2 u_z}{\partial x\partial t}+v^2\frac{\partial^2 u_z}{\partial x^2}\right) \tag{3b}$$

The result is time-invariant in the steady-state conditions, because the excitation is a constant force. Consequently, the partial differentiations with respect to t in Eqs. (3) tend to be zero ($\partial/\partial t=0$, $\partial^2/\partial t^2=0$). Thus, the governing equations of motion for the steady-state response of an elastic body in the moving coordinate system can be written as follows:

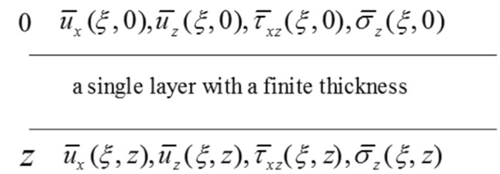

Fig. 2. Stresses and displacements of a single layer with a finite thickness.

$$c_{11}\frac{\partial^2 u_x}{\partial x^2}+c_{44}\frac{\partial^2 u_x}{\partial z^2}+(c_{13}+c_{44})\frac{\partial^2 u_z}{\partial x\partial z}=\rho v^2\frac{\partial^2 u_x}{\partial x^2} \tag{4a}$$

$$(c_{13}+c_{44})\frac{\partial^2 u_x}{\partial x\partial z}+c_{33}\frac{\partial^2 u_z}{\partial z^2}+c_{44}\frac{\partial^2 u_z}{\partial x^2}=\rho v^2\frac{\partial^2 u_z}{\partial x^2} \tag{4b}$$

3. The analytical layer-element solutions for a multilayered half-plane in the moving system

To deal with the problem of a transversely isotropic multilayered half-plane subjected to a time-harmonic load, Ai and Zhang [7] took a Fourier transform to its governing equations of motion, and led to the following equations:

$$\left(\rho\omega^2-\xi^2c_{11}+c_{44}\frac{d^2}{dz^2}\right)\bar{u}_x-(c_{13}+c_{44})\xi\frac{d\bar{u}_z}{dz}=0 \tag{5a}$$

$$(c_{44}+c_{13})\xi\frac{d\bar{u}_x}{dz}+\left(\rho\omega^2-\xi^2c_{44}+c_{33}\frac{d^2}{dz^2}\right)\bar{u}_z=0 \tag{5b}$$

where ω is the circular frequency of the time-harmonic load.

Similarly, in order to solve the partial differential equations (4), we may apply the same Fourier transform in Ref. [7] to reduce them into ordinary differential equations as follows:

$$\left(\rho v^2\xi^2-\xi^2c_{11}+c_{44}\frac{d^2}{dz^2}\right)\bar{u}_x-(c_{13}+c_{44})\xi\frac{d\bar{u}_z}{dz}=0 \tag{6a}$$

$$(c_{44}+c_{13})\xi\frac{d\bar{u}_x}{dz}+\left(\rho v^2\xi^2-\xi^2c_{44}+c_{33}\frac{d^2}{dz^2}\right)\bar{u}_z=0 \tag{6b}$$

According to Eqs. (5) and (6), if $\rho v^2\xi^2$in Eqs. (6) is replaced by $\rho\omega^2$, the two equations will be the same. Therefore, the algebraic operations in Ref. [7] can be used as reference to get the analytical layer-element solutions for a finite layer and a half-pane in the moving system.

After a series of derivation process similar to Ref. [7], the analytical layer-element solutions for a finite layer in the moving coordinate system is established as

$$\begin{bmatrix}-\bar{\boldsymbol{V}}(\xi,0)\\ \bar{\boldsymbol{V}}(\xi,z)\end{bmatrix}=\boldsymbol{K}(\xi,z)\begin{bmatrix}\bar{\boldsymbol{U}}(\xi,0)\\ \bar{\boldsymbol{U}}(\xi,z)\end{bmatrix} \tag{7}$$

where $\bar{\boldsymbol{V}}(\xi,z)=\left[\bar{\tau}_{xz}(\xi,z),\bar{\sigma}_z(\xi,z)\right]^{\mathrm{T}}$,$\bar{\boldsymbol{U}}(\xi,z)=\left[\bar{u}_x(\xi,z),\bar{u}_z(\xi,z)\right]^{\mathrm{T}}$. In addition, $\boldsymbol{K}(\xi,z)$ is a symmetric matrix of order 4×4, which is the analytical layer-element associating the displacements and stresses of $z=0$ and arbitrary depth z in the Fourier transformed domain shown in Fig. 2. The specific elements of the matrix are consistent with Appendix A in Ref. [7] after $\rho\omega^2$ being replaced by $\rho v^2\xi^2$.

Besides, the relationship between displacements and stresses of a half-plane in view of the regularity condition for $z\to\infty$ can also be established:

$$\left[-\bar{\boldsymbol{V}}(\xi,z)\right]=\boldsymbol{K}_h(\xi,z)\left[\bar{\boldsymbol{U}}(\xi,z)\right] \tag{8}$$