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Multi-transmitting formula for finite element modeling of wave propagation in a saturated poroelastic medium

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ABSTRACT

An absorbing boundary condition that is called the multi-transmitting formula (MTF) was originally proposed by Liao and Wong for wave propagation in an elastic medium. In this paper, the MTF was extended for modeling wave propagation in a saturated poroelastic medium using the finite element method. Reflection coefficients of the MTF that is applied to the boundaries of a finite element grid were analytically derived for the incidence of SV, P1 and P2 waves, respectively. The effects of the excitation frequency and the artificial wave velocity on the reflection coefficients of the MTF were theoretically investigated. The MTF was then implemented into a finite element code to examine its capacity to absorb the one-dimensional longitudinal/shear wave, the plain-strain waves, the moving-load generated waves and the three-dimensional waves in the saturated poroelastic medium. It is found that the reflection coefficients evaluated from the numerical simulation agree with the predicted values in the theoretical investigations.

1. Introduction

Due to the semi-infinite nature of the soil, the dynamic analysis of the soil via direct methods, such as finite element and finite difference, requires artificial boundaries to make the computational domain finite. The artificial boundary conditions should allow the wave motion to pass through and generate little or no reflections back into the finite computational domain. Such boundary conditions can be absorbing, transmitting, non-reflecting or silent boundary conditions. Absorbing boundary conditions, which are local both in time and space, have attracted a lot of attention because they can be easily implemented in a finite element or finite difference analysis and can be employed to study nonlinear problems using time step integration techniques.

Several well-known local absorbing boundary conditions have been proposed to transmit elastic waves in an elastic medium, such as the viscous boundary of Lysmer and Kuhlemeyer [1], the viscous-spring boundary of Deeks and Randolph [2], the para-axial boundary of Engquist and Majda [3] and Clayton and Engquist [4], the superposition boundary of Simth [5], the multidirectional boundary of Higdon [6], the high-order non-reflecting boundary condition of Givoli and Neta [7], the infinite element of Bettess and Zienkiewicz [8], the multi-transmitting formula (MTF) of Liao and Wong [9] and the optimal absorbing boundary by Peng and Toksoz [10]. Amongst these, the MTF and the optimal absorbing boundary stand out since they are formulated on a discrete finite element/difference grid and are easy to implement in the finite element/difference analysis. The optimal absorbing boundary condition is constructed by properly locating the zeroes and poles of the reflection coefficients. However, the boundary coefficients are frequency dependent, which hinders its application in time-dependent problems.

The MTF is established with the aim of simulating the general transmitting process of the apparent plane wave propagating along a line normal to the artificial boundary. The MTF involves only one-dimensional stencils, hence it can be applied directly to the two- and three-dimensional cases and, particularly, no special treatment is needed at the interface of the material stratification and at the corners of the computational grid. Ideally speaking, each outgoing plane wave requires an apparent velocity along the line normal to the boundary. However, the incident directions of the waves are not available in advance and the physical velocities are different for each elastic wave type (e.g. S and P waves), which results in all different apparent velocities along the line normal to the boundary. To overcome this, the developers of MTF [9] introduced a common artificial wave velocity c_a to replace all the apparent velocities to formulate the MTF in a compact form; they suggested that c_a be the lowest physical wave velocity of the medium. By introducing several artificial wave velocities instead of

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the single wave velocity c_a in the space-time extrapolation, Liao [11] generalized the proposed MTF to consist of a system of non-reflection boundary conditions. A similar generalization of the MTF was achieved by Wagner and Chew [12] after optimizing the absorption of waves at specific angles of wave incidence. However, the numerical simulations of Wagner and Chew [12] and Liao [13] suggest that the generalization has little effect on the solution error when compared to the original MTF (i.e. simulating the apparent wave propagation process by employing a single artificial wave velocity).

The absorbing boundary conditions developed for the elastic medium have been extended to a saturated poroelastic medium in the literature. According to Biot's theory [14], the interaction between the solid skeleton phase and the pore fluid phase results in three types of dispersive body waves, namely the P1, P2 and S waves, which makes the derivation of absorbing boundaries more difficult. Based on the one-dimensional seismic wave incidence, Zienkiewicz [15] derived a viscous boundary for the $\mathbf{u}-p$ formulation of Biot's theory. At the expense of spurious reflections for oblique incident waves, Degrande and De Roeck [16] developed a viscous boundary for the $\mathbf{u}-\mathbf{w}$ formulation in the frequency domain, which is local in space while non-local in time. By neglecting the P2 wave at the boundary, Akiyoshi et al. [17] and Modaressi and Benzenati [18] proposed viscous boundaries for the $\mathbf{u}-\mathbf{w}$, $\mathbf{u}-\mathbf{U}$ and $\mathbf{u}-p$ formulations. The viscous boundary conditions may suffer from stability problems when dealing with low-frequency dynamic loads. The stability can be improved by adding spring element to the viscous boundary conditions [19], which takes the geometric attenuations of the wave field into consideration. By taking the asymptotic decay of the field quantity into account, Nenning and Schanz [20] formulated an infinite element for unbounded saturated porous media, which is shown to be sufficiently accurate. However the shape functions of the infinite element are constructed in the Laplace domain, and thus the infinite elements are non-local in time. Zhang [21] applied Smith's superposition boundary to the $\mathbf{u}-\mathbf{U}$ formulation of a two-dimensional saturated soil medium; however, this boundary condition can be very expensive in three-dimensional analysis. Gajo et al. [22] formed two types of multidirectional boundaries for the $\mathbf{u}-\mathbf{U}$ formulation at null and infinite permeability, respectively. A preliminary judgment is needed before the proper boundary type can be selected. Zeng et al. [23] extended the perfectly matched layer (PML), which was originally proposed by Berenger [24] as a material absorbing boundary condition for electromagnetic waves, to simulate seismic wave propagation in poroelastic media. Recently, Li and Song [25] derived a high-order transmitting boundary for cylindrical elastic wave propagation in infinite saturated porous media governed by the $\mathbf{u}-p$ formulation of Biot's theory with an assumption of zero permeability. The essence of the proposed transmitting boundary condition is to associate a set of spring, dashpot and mass elements to the boundary nodes. It is worth noting that absorbing boundary conditions involving spring elements or infinite elements need to approximate the decay of the field quantity as a function of r (r is the distance measured to the applied load). For the stationary load inputs, the distance r from the boundary node to the applied load can be easily measured, while r is hard to determine for moving load inputs such as in simulations of ground vibrations caused by high-speed trains.

As it deals only with the displacements at grid points, the MTF can be directly extended to the saturated porous medium without simplification to Biot's theory (e.g. the neglecting of P2 wave, null or infinite permeability). In the following, the MTF, in a discretized form, is proposed for the finite element modeling of wave propagation in a saturated poroelastic medium. Firstly, the reflection coefficients of the MTF, to an arbitrary order N, are analytically derived in the frequency domain for the incidence of P1, P2 and SV waves to the artificial boundary truncating the finite element grid. Parametric studies are performed with respect to the excitation frequency and the artificial wave velocity to investigate their influence on the reflection coefficient of the MTF. Then a finite element code incorporating the second order MTF is developed to investigate the one-, two- and three-dimensional transient wave propagations in the saturated medium that are generated by the displacement and force inputs. The one-dimensional numerical analysis reveals that the computational evaluations of the reflection coefficients of the MTF agree with the predicted values in the theoretical investigations. Specifically, the absorbing effect of MTF is computationally examined for wave propagations in a saturated poroelastic medium generated by the moving load.

2. Theory

A uniform finite-element grid in the $x-z$ plane is shown in Fig. 1. The computational domain is $\{(x,z) : z \le 0\}$ and is occupied by the saturated poroelastic medium. The MTF is applied along the x axis ($z=0$) and the z axis is normal to the artificial boundary. The grid size is Δs and the time step length is Δt.

2.1. MTF formulation

For the boundary point o in Fig. 1, the MTF extrapolates its displacement at time $t=(n+1)\Delta t$ as a linear combination of the displacements at previous time steps along a straight line normal to the artificial boundary in the grid's interior. Let $u_i(j\triangle s, -k\triangle s, n\triangle t)$ denote the soil skeleton displacements of the grid point at $(x=j\Delta s, z=-k\Delta s)$ and at time $t=n\Delta t (i=x,z; j=0, \pm 1, \pm 2, \ldots; k,n=0,1,2,\ldots)$. Similarly, $w_i(j\Delta s, -k\Delta s, n\Delta t)$ denotes the average pore fluid displacement relative to the soil skeleton at the grid point (j,k) and at time $n\triangle t$. The MTF is used to transmit displacements of the soil skeleton and the pore fluid at point o respectively:

$$\begin{cases} u_i(0,0,(n+1)\Delta t) = \sum_{k=1}^{N} (-1)^{k+1} C_k^N u_i(0, -kc_a\Delta t, (n+1-k)\Delta t) \\ w_i(0,0,(n+1)\Delta t) = \sum_{k=1}^{N} (-1)^{k+1} C_k^N w_i(0, -kc_a\Delta t, (n+1-k)\Delta t) \end{cases}, \quad (i=x,z) \tag{1}$$

where, c_a is an artificial wave velocity representing the apparent velocities of waves propagating along the z axis; $C_k^N = N!/[k!(N-k)!]$ is the binomial coefficient and N is the order of the boundary condition.

The computational points in Eq. (1) are located at $z=-kc_a\Delta t$, which do not generally coincide with the grid points in Fig. 1. The quadratic interpolation scheme given by Liao [11] is employed to express the displacement at the computational points in terms of

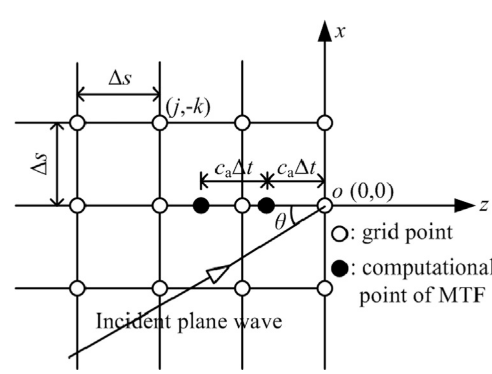

Fig. 1. Schematic view of the uniform finite element grid in the $x-z$ plane and the MTF boundary condition applied at $z=0$.