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Propagation of torsional surface waves in a double porous layer lying over a Gibson half space

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ABSTRACT

The present study deals with the behavior of torsional surface waves when they propagate through an inhomogeneous fluid saturated porous double layers over a dry sandy inhomogeneous Gibson half space. The inhomogeneities of the porous layers are taken as quadratic and exponential variation with depth in the density, elastic moduli and initial stress. In the half space it varies linearly in the elastic moduli and initial stress. For simplicity of the problem, we have used the separation of variable technique. The dispersion equation has been derived with boundary conditions and solved by an iterative method (Newton Raphson method). We have also converted our dynamical equations into non-dimensional form. It has been observed from the numerical validation of the proposed model that the presence of the initial stress and inhomogeneity of both media affect significantly the phase velocity of torsional surface waves. The effect of initial stress, inhomogeneity parameters, depth ratio, sandy parameter, Biot's gravity parameter and porosity of the layer on the dimensionless phase velocity of the torsional surface waves are demonstrated graphically with respect to the non-dimensional wave number kH_1 (where k is the wave number and H_1 is the thickness of the second porous layer).

1. Introduction

The earth is a combination of layers which have different material properties. Basically, four main layers are present in the earth viz the crust, mantle, outer core and inner core. The crust is more heterogeneous in comparison to the other layers and from the last two–three decades maximum earthquakes have occurred in the crust only. Physical damage due to earthquakes is possible only through seismic waves. Surface waves have their own importance for the study of seismic waves.

A large amount of information about the propagation of seismic waves is available in Ewing et al. [1], Aki and Richards [2]. Although there is lot of literature available for Rayleigh, Love and Stoneley waves, very little information is available for torsional surface waves. These waves are horizontally polarized but give a twist to the medium in which they propagate. In a Seismogram some disturbances are found in between the arrival of Rayleigh and Love waves. Earlier, sufficient information was not available for these disturbances, and they were termed as 'noise' and ignored in the study of seismic waves. This 'noise' may be observed in the form of torsional surface waves that propagate in the non-homogeneous earth.

As we know that the earth is an initially stressed medium and presence of the initial stresses in the earth layers are the main cause of earthquakes. Formation of the initial stresses inside the earth is possible due to the slow process of creep, pressure due to overburden, gravitational pull and atmospheric pressure, etc. The full description of the effect of initial stresses and gravity field on elastic waves has been given by Biot [3]. Effect of initial stress on wave velocities at the free surface has been studied by Sharma [4].

Theoretical seismology is basically related to the study of seismic wave propagation in layered media. Surface wave propagation over homogeneous and inhomogeneous medium is an important topic in wave theory. First time, Wilson [5] has studied the propagation of surface waves through inhomogeneous elastic media.

The basic application of mathematical modelling in elasticity theory has been first given in the book of Love [6]. Information related to the application of mathematics in seismology is available in the book of Bath [7] and Bell [8]. A good amount of information related to propagation of seismic waves in the different earth layers is contained in the book by Achenbach [9]. We know that the torsional surface waves has a decreasing trend with respect to the depth of the layer from the surface. Also, the composition of the earth is inhomogeneous and it contains hard as

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well as soft layers (sandy layers). A dry sandy layer is nothing but a layer that consists of sandy particles and the effect of a gravitation field is possible on these particles [3]. With the application of dry sandy layer in the study of *SH* wave, Sharma and Gogna [10] have solved an interface problem and have given some important results for the reflection and transmission of *SH* wave. Very few studies have been carried out on the propagation of torsional surface waves in a sandy medium of which some of them are [11–13].

The dynamical behavior of a fluid saturated porous media has great importance in many fields, such as seismology, earthquake engineering, soil dynamics and fluid dynamics. A theory of the propagation of elastic waves in fluid saturated porous media in the low frequency range has been first studied by Biot [14]. With respect to the Biot's theory, a lot of literature is available on wave propagation in a fluid saturated porous medium in which some of them are [15–22]. Recently, Pal and Ghorai [23] have published a paper on propagation of Love waves at the interface between dry sandy layer and porous half space in which they have shown the effect of gravity parameter, porosity and sandy parameter on phase velocity. Because of its importance in these fields, the study of torsional surface waves has seen considerable attention by researchers in recent years. Many of the implications and applications of this medium in different research fields have been studied by researchers. Notable among these are [24–26].

The present study is based on the propagation of torsional surface waves in a three layers media in which two layers are two different porous solids and remaining one is the dry sandy Gibson half space. In this study, we try to determine the existence of torsional surface waves in an inhomogeneous fluid saturated second porous layer lying in between an inhomogeneous fluid saturated first porous layer and an inhomogeneous dry sandy Gibson half space under initial stresses. Further, we assume that the interfaces between these three layers are impervious. Based on the previous available literature it is clear that in the crust different inhomogeneities are available from place to place. As, Bullen [27] has found that the density varies with increase of depth within the earth, this may be possible due to the presence of inhomogeneity of the layers. In another study, Birch [28] has shown that the rigidity of the earth layers varies at different rates with respect to depth. With the view of these two studies, we assume that the inhomogeneity varies in both of the porous layers as quadratically and exponentially in rigidity, density and initial stress, and in the Gibson half space, inhomogeneity varies linearly in rigidity and initial stress [13]. A numerical example is considered, namely a water saturated limestone layer sandwiched in between a kerosene saturated sandstone and a dry sandy half space (Gibson half space). We discuss the effect of an initial stress P_{20} in the second layer, inhomogeneity variation δ/k, Biot's gravity parameter G', sandy parameter η, width ratio (H_2/H_1) and porosity of the second layer in terms of d_2 on the phase velocity (c/c_2) of the torsional surface waves. These results are discussed in the result and discussion section with respect to the corresponding figures.

2. Statement of the problem

Consider the study of torsional surface waves in an inhomogeneous anisotropic fluid saturated porous layer (water saturated limestone) with thickness H_1 lying in between the dry sandy Gibson half space and the inhomogeneous anisotropic fluid saturated porous layer (kerosene saturated sandstone) with thickness H_2 with varying initial stresses in the radial direction (*r*-axis). The geometry of the present problem has been shown in Fig. 1. The cylindrical polar co-ordinate system has been taken for the present study in which *z*-axis pointing vertically downward as shown in Fig. 1. The radial direction (*r*-axis) pointing in the direction of the wave propagation and the interface between the middle layer and half space has been located at $z=0$.

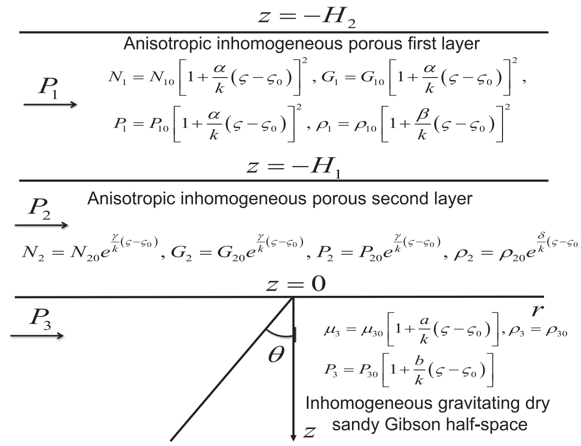

Fig. 1. Geometry of the problem.

2.1. For inhomogeneous porous layers

The dynamical equation of an initially stressed poro-elastic medium can be obtained by Biot's dynamical equations (neglecting the viscosity of the liquid and body forces) for a porous layer under compressional initial stress as given by Biot [3,14]. Here, we use indices $l=1$ for inhomogeneous first porous layer and $l=2$ for inhomogeneous second porous layer.

$$\frac{\partial s_{rr}^{(l)}}{\partial r}+\frac{1}{r}\frac{\partial s_{r\theta}^{(l)}}{\partial \theta}+\frac{\partial s_{rz}^{(l)}}{\partial z}+\frac{s_{rr}^{(l)}-s_{\theta\theta}^{(l)}}{r}-P_l\left(\frac{\partial \omega_z^{(l)}}{\partial \theta}+\frac{\partial \omega_\theta^{(l)}}{\partial z}\right)=\frac{\partial^2}{\partial t^2}\left(\rho_{rr}^{(l)}u_r^{(l)}+\rho_{r\theta}^{(l)}U_r^{(l)}\right),$$

$$\frac{\partial s_{r\theta}^{(l)}}{\partial r}+\frac{2}{r}s_{r\theta}^{(l)}+\frac{1}{r}\frac{\partial s_{\theta\theta}^{(l)}}{\partial \theta}+\frac{\partial s_{\theta z}^{(l)}}{\partial z}-P_l\frac{\partial \omega_z^{(l)}}{\partial r}=\frac{\partial^2}{\partial t^2}\left(\rho_{rr}^{(l)}u_\theta^{(l)}+\rho_{r\theta}^{(l)}U_\theta^{(l)}\right),$$

$$\frac{\partial s_{rz}^{(l)}}{\partial r}+\frac{1}{r}\frac{\partial s_{z\theta}^{(l)}}{\partial \theta}+\frac{\partial s_{zz}^{(l)}}{\partial z}+\frac{1}{r}s_{rz}^{(l)}+P_l\frac{\partial \omega_z^{(l)}}{\partial \theta}=\frac{\partial^2}{\partial t^2}\left(\rho_{rr}^{(l)}u_z^{(l)}+\rho_{r\theta}^{(l)}U_z^{(l)}\right) \tag{1}$$

$$\frac{\partial s^{(l)}}{\partial r}=\frac{\partial^2}{\partial t^2}\left(\rho_{r\theta}^{(l)}u_r^{(l)}+\rho_{\theta\theta}^{(l)}U_r^{(l)}\right),$$

$$\frac{\partial s^{(l)}}{\partial \theta}=\frac{\partial^2}{\partial t^2}\left(\rho_{r\theta}^{(l)}u_\theta^{(l)}+\rho_{\theta\theta}^{(l)}U_\theta^{(l)}\right),$$

$$\frac{\partial s^{(l)}}{\partial z}=\frac{\partial^2}{\partial t^2}\left(\rho_{r\theta}^{(l)}u_z^{(l)}+\rho_{\theta\theta}^{(l)}U_z^{(l)}\right) \tag{2}$$

where, $s_{jk}^{(l)}(j,k=r,\theta,z)$ and $s^{(l)}$ are the incremental stress components of solid and stress in liquid, respectively. $\left(u_r^{(l)},u_\theta^{(l)},u_z^{(l)}\right)$ and $\left(U_r^{(l)},U_\theta^{(l)},U_z^{(l)}\right)$ are the components of displacement vectors for solid and liquid, respectively.

The rotational components are given by

$$\omega_r^{(l)}=\frac{1}{2r}\left(\frac{\partial u_z^{(l)}}{\partial \theta}-\frac{\partial u_\theta^{(l)}}{\partial z}\right),\quad \omega_\theta^{(l)}=\frac{1}{2}\left(\frac{\partial u_r^{(l)}}{\partial z}-\frac{\partial u_z^{(l)}}{\partial r}\right),$$

$$\omega_z^{(l)}=\frac{1}{2r}\left(\frac{\partial}{\partial r}(ru_\theta^{(l)})-\frac{\partial u_r^{(l)}}{\partial \theta}\right) \tag{3}$$

The stress-strain relations for the anisotropic fluid saturated porous layers under normal initial stress P_l are

$$s_{rr}^{(l)}=(A_l+P_l)e_{rr}^{(l)}+(A_l-2N_l+P_l)e_{\theta\theta}^{(l)}+(F_l+P_l)e_{zz}^{(l)}+Q_l\varepsilon^{(l)},$$

$$s_{\theta\theta}^{(l)}=(A_l-2N_l)e_{rr}^{(l)}+A_le_{\theta\theta}^{(l)}+F_le_{zz}^{(l)}+Q_l\varepsilon^{(l)},\quad s_{r\theta}^{(l)}=2N_le_{r\theta}^{(l)},$$

$$s_{zz}^{(l)}=F_le_{rr}^{(l)}+F_le_{\theta\theta}^{(l)}+C_le_{zz}^{(l)}+Q_l\varepsilon^{(l)},\quad s_{\theta z}^{(l)}=2G_le_{\theta z}^{(l)},\quad s_{rz}^{(l)}=2G_le_{rz}^{(l)} \tag{4}$$