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Technical note

Variational analysis of dynamics of fissured poroelastic rocks

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ABSTRACT

Rocks can be modeled in a continuum framework as fissured, poroelastic materials, i.e., materials with two degrees of porosity, one due to the fissures and another one due to the pores. The governing equations of motion of fissured poroelastic rocks established by Beskos are rederived here by establishing a variational statement which also provides the boundary conditions of the problem. This is accomplished by considering strain, dissipation and kinetic energies as well as the work of external forces. The above statement is also derived here by employing the method of weighted residuals.

1. Introduction

The determination of the quasi-static or dynamic behavior of fluid-saturated poroelastic media, such as soils and rocks, is an important problem in many practical geomechanics applications. The most widely used theory for fluid-saturated poroelastic soils is that of Biot [1,2]. This theory, which has been successfully applied for the solution of a plethora of practical problems of fluid-saturated poroelastic soils [3], cannot be successfully used for problems involving fluid-saturated poroelastic rocks, which are generally characterized not only by pores but also by fissures.

In fluid-saturated poroelastic rocks one has fissures separating the rock mass in porous blocks. The permeability of the fissures is much higher than that of the porous blocks. On the other hand, the porosity of the porous blocks is much higher than that of the fissures. Using a double porosity model, Wilson and Aifantis [4] and Beskos and Aifantis [5] conducted analytical studies and Khaled et al. [6] numerical studies (using the finite element method – FEM) on the consolidation of fluid-saturated, fissured, poroelastic rocks.

Other double porosity models for fluid-saturated, fissured, poroelastic rocks were developed and used for consolidation analysis with the aid of analytical or FEMs by Valliappan and Khalili-Naghadeh [7], Elsworth and Bai [8], Bai et al. [9], Auriault and Boutin [10,11], Berryman and Wang [12], Lewallen and Wang [13] and Bai et al. [14].

Inclusion of inertia effects in quasi-static (consolidation) double porosity models enables one to study wave propagation problems in fluid-saturated, fissured, poroelastic rocks. Here one can mention the works of Beskos [15], Beskos et al. [16,17], Vgenopoulou and Beskos [18], Auriault and Boutin [19], Berryman and Wang [20] and Auriault [21].

Most of the aforementioned works on quasi-static (consolidation) problems and all on dynamic problems involving double porosity models for rocks are analytical. However, realistic boundary value problems can only be solved by numerical methods, such as the FEM. Finite elements have been used for the solution of consolidation problems of fluid-saturated, fissured poroelastic rocks in [6,7,9] and [14]. In all these cases the method of weighted residuals-Galerkin has been employed for the formulation of the problem, in accordance with the work of Zienkiewicz and Shiomi [22] for the dynamics of single porosity media. Another approach could be to construct and employ a variational statement like those used by Ghaboussi and Wilson [23,24] for the quasi-static and dynamic behavior of single porosity media.

In general, variational statements for a set of governing partial differential equations can recover those governing equations and also provide all possible combinations of boundary conditions, which are not easy to establish in coupled fluid-deformation problems. In addition, they can also be used for formulating the finite element equations in matrix form. Finally, variational statements can be successfully used to prove uniqueness, as it was done in Beskos and Aifantis [5] for the case of consolidation of double porosity media. The method of weighted residuals or Galerkin approach, as it is known in FEMs, has the advantage of deriving the matrix finite element equations in an easier way than variational statements. On the other hand, one can recover variational statements with the aid of the method of weighted residuals as demonstrated in Papargyri-Beskou et al. [25] for gradient elastic beams under static or dynamic loads.

In the present paper, the governing equations of motion for fluid-saturated, fissured, poroelastic rocks developed by Beskos

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[15] are recovered by establishing a variational statement on the basis of strain, kinetic and dissipation energies of a double porosity medium. This variational statement also serves to establish all possible boundary conditions. In addition, the variational statement is derived with the aid of the method of weighted residuals as applied to the governing equations of motion in [15].

2. Dynamic behavior of fissured poroelastic rocks

The governing equations of motion of fissured poroelastic rocks, as obtained by Beskos [15] on the basis of the theory of mixtures, have the form

$$(\lambda+\mu)u_{j,ji}+\mu u_{i,jj}=-\overline{v}_1\overline{Q}_i^1-\overline{v}_2\overline{Q}_i^2+\overline{\rho}_S\ddot{u}_i;\quad i,j,k=1,2,3 \tag{1}$$

$$-\beta_a P_{a,i}=\overline{v}_a\overline{Q}_i^a+\rho_f\dot{\overline{Q}}_i^a+\overline{\rho}_a\ddot{u}_i;\qquad a=1,2 \tag{2}$$

$$\beta_a\dot{u}_{i,i}+a_a\dot{P}_a+\overline{Q}_{i,i}^a=-(-1)^a\kappa(P_2-P_1);\qquad a=1,2 \tag{3}$$

In the above, u_i are the displacement components of the solid elastic skeleton, the subscript or superscript α stands for the fluid in the fissures ($\alpha=1$) and the pores ($\alpha=2$), P_a denotes fluid pressure, $\overline{Q}_i^a$ stands for the relative specific discharge of the form

$$\overline{Q}_i^a=n_a(\dot{u}_i^a-\dot{u}_i)=\dot{w}_i^a \tag{4}$$

with n_α and u_i^a denoting porosity and fluid displacement in the fissures ($\alpha=1$) and pores ($\alpha=2$), $\overline{v}_a$ is defined as

$$\overline{v}_a=\beta_a\frac{v}{k_a} \tag{5}$$

where v is the dynamic viscosity of the fluid, k_a the permeability and β_a coefficients expressing the effect of the solid deformability on the fluid flow, $\overline{\rho}_s$ and $\overline{\rho}_a$ are relative densities of the solid and fluid, respectively, of the form

$$\overline{\rho}_s=\rho_s[1-(n_1-n_2)],\ \overline{\rho}_a=n_a\rho_f \tag{6}$$

with ρ_s and ρ_f being the mass densities of solid and fluid, respectively, and λ and μ are the Lamé constants of the elastic solid, a_a measure the compressibilities of fissures ($\alpha=1$) and pores ($\alpha=2$) filled with fluid and κ measures the transfer of fluid from the pores to the fissures.

Furthermore, summation convention is assumed over repeated indices, commas indicate differentiation with respect to space variables and overdots denote differentiation with respect to time t. Eqs. (1)–(3) form of system of $3+2\times 3+2=11$ partial differential equations with 11 unknowns, i.e., three u_i, three $\overline{Q}_i^1$, three $\overline{Q}_i^2$, P_1 and P_2. It should be noted that the coefficient of$\dot{u}_{i,i}$ in Eq. (3) is here just β_a and not β_a+n_a as it is in [15] in order to be compatible with the same equation in references [5,6], which, even though dealing with the quasi static case and not the dynamic one, are also utilized here.

3. Variational principle

Consider the energy functional L given by

$$L=U+D-K-W \tag{7}$$

where U is the strain energy of the solid and the fluids in fissures and pores, D is the dissipation energy due to the friction of the fluid flows, K is the kinetic energy of the solid and the fluid in fissures and pores and W is the work of external forces at the boundary S of the solid–fluid body of volume V. Following [5], one has that

$$U=\int_V\left(\frac{1}{2}\sigma_{ij}^s\epsilon_{ij}+\frac{1}{2}\sum_{\alpha=1}^{2}\alpha_\alpha P_a^2+\frac{1}{2}\frac{1}{s}\kappa(P_2-P_1)^2\right)dV \tag{8}$$

$$D=\sum_{a=1}^{2}D_a=\frac{1}{2}s\sum_{a=1}^{2}\int_V\frac{\nu}{k_a}(w_i^a)^2dV \tag{9}$$

where $s=\partial/\partial t$ and ε_{ij} and σ_{ij}^s are the strain and stress tensors of the elastic solid, respectively, of the form

$$\varepsilon_{ij}=\frac{1}{2}(u_{i,j}+u_{j,i}),\ \sigma_{ij}^s=\lambda u_{k,k}\delta_{ij}+\mu(u_{i,j}+u_{j,i}) \tag{10}$$

with the δ_{ij} being the Kronecker's delta. The kinetik energy K can be expressed as

$$K=K^S+\sum_{a=1}^{2}K^a=\int_V\frac{1}{2}\overline{\rho}_S(\dot{u}_i)^2dV+\sum_{a=1}^{2}\int_V\frac{1}{2}\overline{\rho}_a(\dot{u}_i^a)^2dV \tag{11}$$

while the work of the external forces as

$$W=\int_S S_i^t u_i ds+\sum_{a=1}^{2}\int_S s_i^a w_i^a ds \tag{12}$$

where S_i^t is the total traction vector and s_i^a the fluid pressure vector of the form

$$S_i^t=\sigma_{ij}^t\eta_j=\left(\sigma_{ij}^s-\sum_{a=1}^{2}\beta_a p_a\right)\eta_j \tag{13}$$

$$s_i^a=-p_a\delta_{ij}\eta_j \tag{14}$$

with η_j being the unit normal vector on the boundary surface S.

Employing Hamilton's principle, the variation of L as given by Eqs. (7)–(9), (11) and (12) takes the form

$$\begin{aligned}\delta L=\delta\int_t dt\Bigg\{\int_V\Bigg(&\frac{1}{2}\sigma_{ij}^S\epsilon_{ij}+\frac{1}{2}\sum_{a=1}^{2}\alpha_a P_a^2+\frac{1}{2}\frac{1}{s}\kappa(P_2-P_1)^2\\&+\frac{1}{2}s\sum_{a=1}^{2}\frac{\nu}{k_a}(w_i^a)^2\Bigg)dV\Bigg\}-\delta\int_t dt\left\{\int_V\frac{1}{2}\overline{\rho}_S(\dot{u}_i)^2dV\right\}\\&-\delta\int_t dt\left\{\int_V\frac{1}{2}\overline{\rho}_1(\dot{u}_i^1)^2dV\right\}-\delta\int_t dt\left\{\int_V\frac{1}{2}\overline{\rho}_2(\dot{u}_i^2)^2dV\right\}\\&-\delta\int_t dt\left\{\int_S s_i^t u_i ds\right\}-\delta\int_t dt\left\{\sum_{a=1}^{2}\int_S s_i^a w_i^a ds\right\}\end{aligned} \tag{15}$$

or after the use of Eq. (10) and Green's theorem

$$\begin{aligned}\delta L=&\iint_{V,t}-\{(\lambda+\mu)u_{j,ji}+\mu u_{i,jj}\}\delta u_i dV dt+\iint_{S,t}(\lambda u_{k,k}\delta_{ij}\\&+\mu(u_{i,j}+u_{j,i})\eta_j\delta u_i ds dt+\iint_{V,t}\sum_{a=1}^{2}a_a P_a\delta P_a dV dt\\&+\iint_{V,t}\frac{1}{s}k(P_2-P_1)\delta(P_2-P_1)dV dt+\sum_{a=1}^{2}s\frac{\nu}{k_a}\iint_{V,t}w_j^a\delta w_j^a dV dt\\&-\int_V\left[\overline{\rho}_S\dot{u}_i\delta u_i+\sum_{a=1}^{2}\overline{\rho}_a\dot{u}_i^a\delta u_i^a\right]_{t_0}^{t_1}dV\\&+\iint_{V,t}\overline{\rho}_S\ddot{u}_i\delta u_i dV dt+\iint_{V,t}\sum_{a=1}^{2}\overline{\rho}_a\ddot{u}_i^a\delta u_i^a dV dt\\&-\iint_{S,t}S_i^t\delta u_i ds dt-\sum_{a=1}^{2}\iint_{S_a,t}s_i^a\delta w_i^a ds dt\end{aligned} \tag{16}$$

where $S_i^s=\sigma_{ij}^s\eta_j$.

At this point, the constitutive assumption

$$w_{i,i}^a=-\beta_a u_{k,k}-a_a P_a-(-1)^a\kappa\frac{1}{s}(P_2-P_1)\qquad a=1,2 \tag{17}$$