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Technical Note

Numerical prediction of ground vibrations induced by high-speed trains including wheel–rail–soil coupled effects

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ABSTRACT

A simplified analytical model including the coupled effects of the wheel–rail–soil system and geometric irregularities of the track is proposed for evaluation of the moving train load. The wheel–rail–soil system is simulated as a series of moving point loads on an Euler–Bernoulli beam resting on a visco-elastic half-space, and the wave-number transform is adopted to derive the 2.5D finite element formulation. The numerical model is validated by published data in the literature. Numerical predictions of ground vibrations by using the proposed method are conducted at a site on the Qin-Shen Line in China.

1. Introduction

Ground vibration induced by moving high-speed trains has increasingly received attention in recent years. The dominant frequency of the induced ground vibration ranges from 5 to 150 Hz, which may cause vertical resonance and damage to nearby structures. During the past decades, various analytical, semi-analytical and numerical methods with different accuracies have been developed to investigate train-induced ground vibrations. One of the methods is the two-and-a-half-dimensional finite element method (2.5D FEM). The method is computational effective in that the 3D problem is solved by using a 2D plane model for which the applied load is along the direction out of the plane [1–4]. However, the dynamic loads generated by geometric irregularities of high-speed rail tracks were not taken into account in these studies. In addition, due to the complexity of the dynamic interactions among the train, track and ground, the train is commonly decoupled from the track–soil system and represented by a series of 'static' loads moving with a constant velocity. As a result, the dynamic coupling of the train, the rail and the subgrade cannot be taken into account.

In this paper, a 2.5D FEM wheel–track–soil interaction model in a visco-elastic medium is proposed, where the external moving train load is derived analytically. The proposed method includes both geometric irregularities of the track and the wheel–track–soil coupling effects.

2. Wheel–rail–soil interaction model

Similar to Ref. [4], the track-embankment and ground are respectively modeled as an Euler–Bernoulli beam and a horizontally layered visco-elastic half-space.

The effect of vehicles moving along the ground surface with a velocity c can be represented by a moving load of $\sum_{n=1}^{N} P_n\delta(y-ct-a_n)$ [5]. The moving load $P_n=f(t)$ is derived semi-analytically to incorporate the dynamic interaction of the vehicles, track irregularities and soil.

Considering a quarter vehicle shown in Fig. 1, the equilibrium equations of a single wheel-set can be described as

$$\frac{1}{4}m_c\ddot{u}_c+d_{p2}(\dot{u}_c-\dot{u}_b)+k_{s2}(u_c-u_b)=\frac{1}{4}m_cg\delta(x-ct) \quad (1)$$

$$\frac{m_b}{2}\ddot{u}_b-d_{p2}(\dot{u}_c-\dot{u}_b)-k_{s2}(u_c-u_b)+d_{p1}(\dot{u}_b-\dot{u}_w)+k_{s1}(u_b-u_w)=\frac{1}{2}m_bg\delta(x-ct) \quad (2)$$

$$m_w\ddot{u}_w-d_{p1}(\dot{u}_b-\dot{u}_w)-k_{s1}(u_b-u_w)=\delta(x-ct)[m_wg-f(t)] \quad (3)$$

where m_b is the weight of bogie; m_c is the weight of carriage; m_w is the weight of a single wheel-set; k_{s1} and k_{s2} are the vertical

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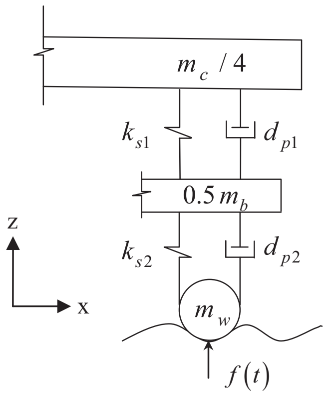

Fig. 1. Analytical model for a single wheel-set.

primary and secondary spring stiffness, respectively; d_{p1} and d_{p2} are the vertical primary and secondary damping coefficients, respectively; $u_b(t)$, $\dot{u}_b(t)$ and $\ddot{u}_b(t)$ are displacement, velocity and acceleration of the bogie, respectively; $u_c(t)$, $\dot{u}_c(t)$ and $\ddot{u}_c(t)$ are displacement, velocity and acceleration of the carriage, respectively; $u_w(t)$, $\dot{u}_w(t)$ and $\ddot{u}_w(t)$ are displacement, velocity and acceleration of a single wheel-set; $f(t)$ is the subgrade reaction; g is acceleration of gravity; and δ is Delta function.

By making the hypothesis that both the vehicle dynamic load caused by rail irregularities and the vibration of the vehicle for harmonic oscillations take the maximum, a new equation in the frequency domain can be derived as

$$f^t(\omega-\xi_x c) = a_8 u_w^{xt} + A_7\delta\left(\xi_x - \frac{\omega}{c}\right) \tag{4a}$$

$$u_w^{xt} = a_7 f^t(\omega-\xi_x c) + A_6\delta\left(\xi_x - \frac{\omega}{c}\right) \tag{4b}$$

in which u_w^{xt} is the displacement of a single wheel-set in the frequency (superscript t) – wave number domain (superscript x); ω is the angular frequency; ξ_x is the x-directional wave-number.; a_7 and a_8 are dynamic-stiffness or flexibility coefficients of the system; A_6 and A_7 are coupling coefficients; $a_7 = a_6/a_5$, $a_8 = 1/\ a_7$; $A_6 = A_5/a_5$, $A_7 = -A_5/a_6$; $a_6 = (a_4/4a_2a_3)m_c\omega^2 + (1/2a_2)\ m_b\omega^2 - 1$, $a_5 = (a_1a_4/4a_2a_3)m_c\omega^2 + (a_1/2a_2)m_b\omega^2 - m_w\omega^2$; $A_5 = A_4\ + (A_2/4a_3)m_c\omega^2 - (a_4/4a_2a_3)m_c\omega^2 A_3 - (1/2a_2)m_b\omega^2 A_3$; $a_1 = -m_w\omega^2 + i\omega d_{p1} + k_{s1}$, $a_2 = i\omega d_{p1} + k_{s1}$, $a_3 = (1/4)m_c\omega^2 + imd_{p2} + k_{s2}$, $a_4 = i\omega d_{p2} + k_{s2}$, $A_4 = A_1 + A_2 + A_3$, $A_1 = (\pi m_b g)/c$, $A_2 = (\pi m_c g)/(2c)$, $A_3 = (2\pi m_w g)/c$; f^t is subgrade reaction in the frequency domain.

Considering the frequency shift properties of the fast Fourier transform (FFT), the moving train load consisting of N group wheel-sets are represented in the time and frequency domains as

$$p(x,y,z,t) = \sum_{i=1}^{N} f(t)\delta(x-ct-a_i) \tag{5a}$$

$$p^{xt} = \sum_{i=1}^{N} f^t(\omega-\xi_x c)e^{-i\xi_x a_i} \tag{5b}$$

Substituting Eq. (4a) into Eq. (5b), the p^{xt} can be written as

$$p^{xt} = \sum_{i=1}^{N}\left(a_8 u_w^{xt} + A_7\delta\left(\xi_x - \frac{\omega}{c}\right)\right)e^{-i\xi_x a_i} \tag{6}$$

Assuming the track irregularity as a harmonic function, described as

$$u_{irr} = A_t e^{i(2\pi/\lambda_t)x} \tag{7}$$

where A_t is the amplitude of the track irregularity; λ_t is the wavelength of the track irregularity. The track geometric irregularity in the frequency-wave number domain can be expressed as Eq. (8), in which $\Omega = 2\pi c/\lambda_t$.

$$u_{irr}^{xt} = \frac{2\pi A_t}{c}\delta\left(\xi_x - \frac{\omega-\Omega}{c}\right) \tag{8}$$

The track-embankment structure is modeled as an Euler–Bernoulli beam sleeping on the ground. The track is along the x-axis and the governing equation of the beam under the moving load $p(x,y,z,t)$ in frequency domain is expressed as

$$(EI\xi_x^4 - m_r\omega^2)u_{rb}^{xt} = p^{xt} - f^{xt}(\xi_x,\omega) \tag{9}$$

where EI is bending stiffness of the two tracks; m_r is mass density of two tracks per unit length; u_{rb}^{xt} and f^{xt} are the track bending displacement and subgrade reaction in the frequency-wave number domain, respectively.

The wheel and rail is considered to be always staying in contact in the process of train moving. The deformation of the wheel and rail meet the compatibility condition in the frequency-wave number domain, described as

$$u_w^{xt} = u_{irr}^{xt} + u_{rb}^{xt} \tag{10}$$

in which u_w^{xt} and u_{irr}^{xt} are a single wheel-set displacement and the track irregularity in the frequency-wave number domain.

Finally, the governing equation can be replaced as

$$\left(EI\xi_x^4 - m_r\omega^2 - \sum_{i=1}^{N} a_8 e^{-i\xi_x a_i}\right)u_r^{xt}$$

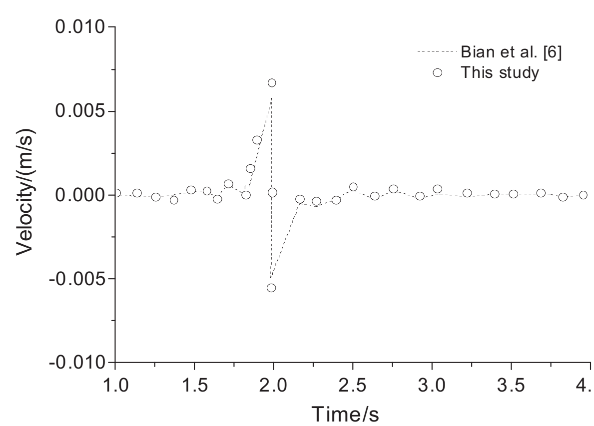

Fig. 2. Computed vertical ground velocity time history at the track center.

Table 1
Soil parameters used in numerical simulations on the Qin-Shen line.

Soil layer	Thickness (m)	Mass density ρ (kg/m^3)	Shear velocity $c_s(m/s)$	P-wave velocity c_p (m/s)	Poisson's ratio υ	Damping ratio β
Equivalent layer for embankment	2.0	2200	170	298	0.25	0.25
Silty clay	2.0	1700	150	270	0.30	0.25
Half-space (fine sand)	26	1800	280	490	0.25	0.25