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Technical Note

# Three-dimensional limit analysis of seismic displacement of slope reinforced with piles

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ABSTRACT

Three-dimensional (3D) limit analysis of seismic stability of slopes reinforced with one row of piles is presented in this paper. A 3D rotational mechanism for earth slope is adopted. The lateral forces provided by the piles are evaluated by the theory of plastic deformation. Expressions for calculating the yield acceleration coefficient are derived. A random iteration method is employed to find the critical acceleration coefficient for the 3D slopes with or without reinforcement. Based on the kinematic theory within the frame of the pseudo-static approach, a 3D model is proposed for evaluating the critical state and the subsequent displacement response. Furthermore, Newmark's analytical procedure is employed to estimate the cumulative displacement induced by given earthquake loads. An example is shown to illustrate the influence of the piles on the seismic displacement of the 3D slopes.

## 1. Introduction

The stability of the slopes subjected to seismic loads has become a serious problem concerned in geotechnical engineering because of the worldwide occurrence of earthquake events in recent years. Earth retaining system, such as piles and anchors, is used to stabilize the active landslides and unstable slopes. The traditional seismic analysis of geotechnical structures considers only the factor of safety but neglects the earthquake shaking process (e.g., [8,25,26]). Design of slope reinforcement using the pseudo-static approach may lead to an unrealistically long reinforcement for large ground accelerations [16,18]. Therefore, a displacement methodology is introduced to reduce the reinforcement length [21].

An analysis based on upper bound limit analysis and [23]) analytical procedure was implemented to assess the soil displacement of two-dimensional (2D) slopes without reinforcement [5]. This method was later adopted to analyze the directed sliding mechanism for geosynthetics reinforced soil structures [16]. Applications of Newmark model to the development of a permanent displacement analysis for geosynthetics reinforced slopes subjected to seismic loads was found in literatures [17,21]. Additionally, the seismic displacement of slopes reinforced with piles was analyzed by [15]. All the literatures mentioned above analyzed 2D plane–strain failure mechanisms. However, it was commonly acknowledged that 2D solutions were conservative to analyze slope stability compared with 3D solutions [4,10]. In order to provide a more accurate prediction of the displacement of the reinforced slopes subjected to seismic loads, 3D analysis of the slopes should be considered.

Michalowski and Drescher [19] have proposed a class of 3D admissible rotational failure mechanisms for slopes (referring to Fig. 1), which definitely promotes the application of the limit analysis method in 3D stability of earth slopes [10]. In this technical note, these 3D failure mechanisms are adopted. Furthermore, the random iteration method [13] is used to find the critical failure surface. The kinematic approach of limit analysis is used to calculate the yield acceleration of the reinforced slope. Considering the seismic loads applied on the slope, the cumulative displacement induced by the earthquake is estimated by the Newmark model.

## 2. Critical acceleration for 3D slopes reinforced with piles

In this study, the soil of the slopes is considered to be homogeneous and isotropic. The failure surface in the 3D slopes is assumed to be the curvilinear cone (the shape is similar to a horn), with upper and lower contours defined by log-spirals [19], as shown in Fig. 1. The shape of the failure surface is smooth, and

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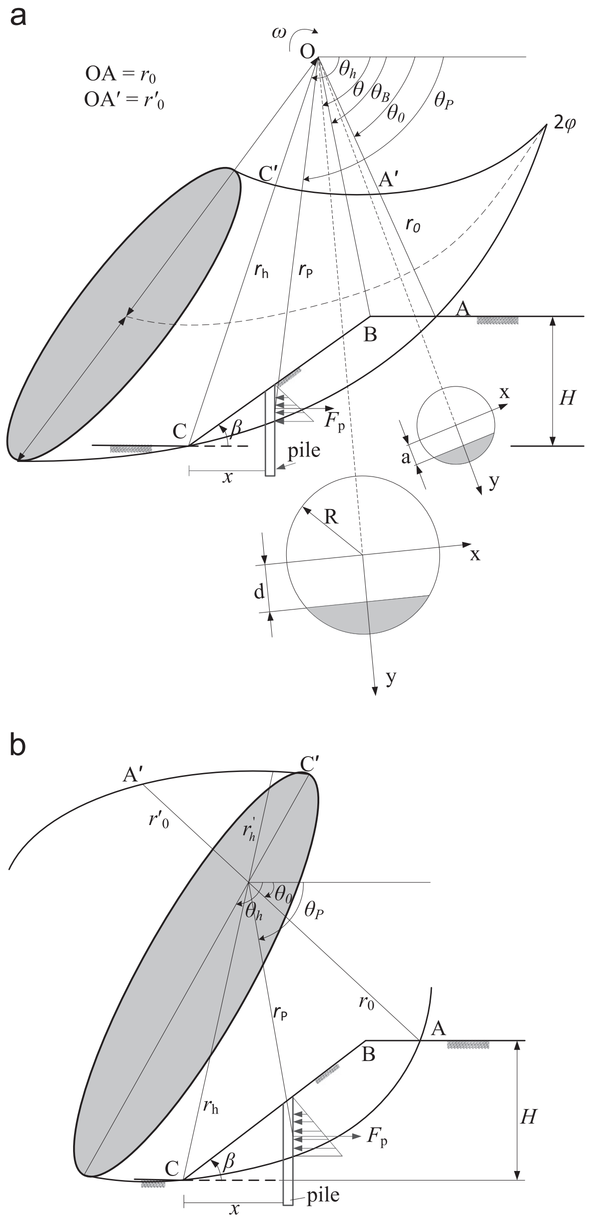

**Fig. 1.** Three-dimensional rotational toe-failure mechanism in stabilized slopes: (a) a 'horn-shape' surface; (b) alternative mechanism (based on [19]).

has one symmetry plane. The trace of the mechanism on the symmetry plane is described by two log-spirals, AC

$$r = r_0 e^{(\theta-\theta_0)\tan\,\varphi} \tag{1}$$

and A′C′

$$r' = r'_0 e^{-(\theta-\theta_0)\tan\,\varphi} \tag{2}$$

where $r_0$ is the radius of the log spiral with respect to angle $\theta_0$, as shown in Fig. 1(a). $\varphi$ is the internal friction angle of the soil. The location of the 'horn' for a specified slope is uniquely determined by the angles $\theta_0$, $\theta_h$, the ratio $r'_0/r_0$, and the resistance force $F_P$ provided by the piles. The failure soil mass rotates as a rigid body about the point O with angular velocity $\omega$. The surface in Fig. 1 (a) can be regard as being generated by rotating a circle of increasing the diameter about an axis passing through point O outside the circle. A different admissible mechanism can be generated when the circle is rotated about its chord, shown as Fig. 1(b). This time, the upper contour A′C′ of the curvilinear surface is defined by the log-spiral

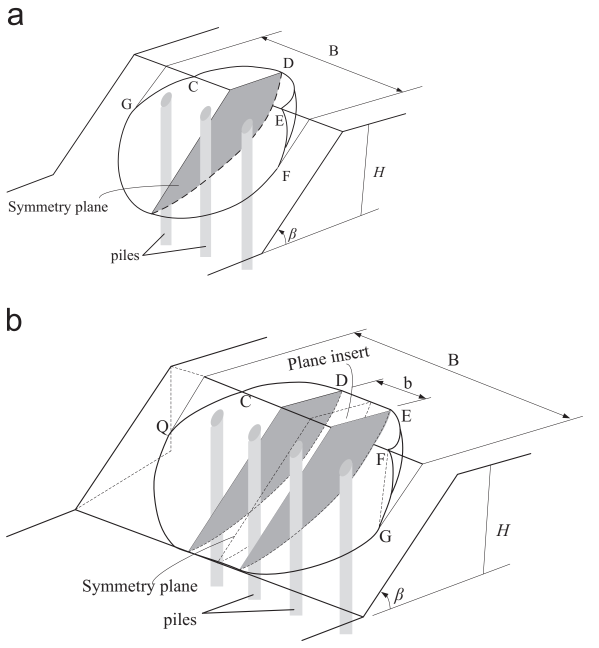

**Fig. 2.** Schematic diagram of 3D rotational failure mechanism with limited width B for slopes stabilized with piles: (a) 3D mechanism; (b) mechanism with plane insert (based on [19]).

$$r' = -r'_0 e^{(\theta-\theta_0)\tan\,\varphi} \tag{3}$$

In order to allow the transition of the 3D failure mechanisms to plane–strain mechanisms, the 3D failure surface model is split from the symmetry plane (Fig. 2(a)), and then separated laterally into two halves. Additionally, an 'insert plane' with a width of $b$ is inserted (Fig. 2(b)). This 'plane insert' modification has been proposed in [19].

The rate of work of the failing soil weight in block CDEFGQ (Fig. 2(b)) during an incipient rotation about point O is calculated as follows:

$$W_\gamma = W_\gamma^{2\mathrm{D}} \cdot b + W_\gamma^{3\mathrm{D}} \tag{4}$$

where the superscript 3D denotes the work rates for the 3D portion of the failure mechanism (block CDEFG in Fig. 2(a)) and 2D relates to the plane insert (Fig. 2(b)). The details of the equation used in calculations are shown in Appendix.

Once the slope is subjected to horizontal shaking, the rate of the inertial force $W_s$ should be considered in the energy balance equation. According to the pseudo-static approach, the horizontal force acting at the center of gravity is calculated as the product of a seismic coefficient $k$ and the weight of potential failing soil mass to represent the effect of the earthquake loading on the failing soil mass. The rate of external work due to the inertial force can be written as follows:

$$\begin{aligned} W_s &= W_s^{2\mathrm{D}} \cdot b + W_s^{3\mathrm{D}} \\ &= k\gamma r_0^3 \omega b (f_1^s - f_2^s - f_3^s) + 2\omega\gamma k f_4^s \end{aligned} \tag{5}$$

where the superscript has the same meaning with Eq. (4), $k$ is the seismic coefficient, $\gamma$ is the unit weight of the soil. The coefficient $f_1^s \sim f_4^s$ can be found in Appendix.

Considering the resistance provided by the piles, the total energy dissipation rate $D$ is the sum of $D_c$ and $D_p$, shown below

$$\begin{aligned} D &= D_c + D_P \\ &= D^{2\mathrm{D}} \cdot b + D^{3\mathrm{D}} + D_P \end{aligned} \tag{6}$$

where $D_p$ is the dissipation rate induced by the reinforcement, $D_c$