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Dispersion of shear wave propagating in vertically heterogeneous double layers overlying an initially stressed isotropic half-space

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ABSTRACT

The present paper investigates the propagation of horizontally polarised shear wave in distinct vertically heterogeneous double layers overlying an isotropic half-space under horizontal initial stress. The vertical heterogeneity in the uppermost layer is caused due to quadratic variation only in rigidity, whereas vertical heterogeneity in the sandwiched layer is caused due to exponential variation in rigidity and density both. The closed form of velocity equation is obtained which leads to the dispersion equation as its real part and damping equation as its imaginary part. The validation of dispersion relation with the classical case is made by using Debye asymptotic expansion which is the major highlight of this study. The significant effect of the width ratio of the layers, heterogeneity parameters of both the layers and horizontal compressive/tensile initial stress on the phase velocity and damped velocity of SH-wave have been traced out. The comparative study and some important peculiarities have been revealed by means of graphical illustrations.

1. Introduction

The crust is relatively more heterogeneous than mantle, which makes the study of wave propagation much practical considering the heterogeneous layers. There are different sort of vertical heterogeneity persist in crustal layers in the form of exponential function, linear function, quadratic function etc. The study of wave propagation in layered elastic media with different boundaries helps to understand and predict the seismic behaviour at the different margins of earth, which makes it applicable in the field of geophysics, civil, mechanical, and other engineering branches. Many researchers had widely studied the theory of Love wave propagation in a medium where the velocity, rigidity and density are functions of depth. Kar [1] worked on the propagation of Love-type waves in a non-homogeneous internal stratum of finite thickness lying between two semi-infinite isotropic media. Love waves in different heterogeneous layered media were studied by Gogna [2]. Scattering of SH-waves in multi-layered media with irregular interfaces have been discussed by Ding and Dravinski [3]. Chattopadhyay et al. [4] discussed the propagation of shear waves in viscoelastic medium at irregular boundaries. Chattopadhyay et al. [5] described the effects of point source and heterogeneity on the propagation of SH-waves. Guz [6] has analysed the three-dimensional linearised theory of elastic waves propagating in initially stressed solids. He formulated surface waves along planar and curvilinear boundaries and interfaces, waves in layers and cylinders, waves in composite materials, waves in hydroelastic systems, and dynamic problems for moving loads. Chattopadhyay [7] discussed the propagation of SH-waves in a sandwiched heterogeneous layer lying between two semi-infinite homogeneous elastic media where the heterogeneity in the sandwiched layer was taken in the form of linearly varying function of depth in the rigidity and density was kept constant. Bhattacharya [8] discussed the possibility of the propagation of Love-type waves in an intermediate heterogeneous layer where the inhomogeneity was assumed in two different forms; the first form dealt with the exponential variation in rigidity and the second was a linear variation in both rigidity and density. Dutta [9,10] discussed two problems relating to the propagation of Love-type waves in a non-homogeneous internal stratum lying between two semi-infinite homogeneous elastic media.

Due to the presence of many physical factors, a large amount of initial stress evolves in a medium which have a pronounced influence on the propagation of waves as shown by Biot [11]. These factors may be overburden layer, variation in temperature, slow process of creep, gravitational field, etc. The Earth is a highly initially stressed medium. Dey and Addy [12] have shown the effect of initial stresses on the propagation of Love waves by considering the layer and the half-space to be isotropic elastic in one case and visco-elastic in another case. Gupta [13] studied the propagation of Love waves in a non-homogeneous substratum

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over an initially stressed heterogeneous half-space. Propagation of Love waves in a non-homogeneous orthotropic elastic layer under changeable initial stress overlying semi-infinite medium was given by Abd-Alla and Ahmed [14].

Keeping in mind, the existence of different types of heterogeneity in the crustal layers and motivated by the fact that earth is an initially stressed body, we considered the present problem with a distinct geometry and heterogeneity configuration not attempted till now. This problem studies the SH-waves propagating in double layers of finite width having different sort of heterogeneity overlying an initially stressed isotropic half-space. The closed form of velocity equation is obtained which leads to the dispersion equation as its real part and damping equation as its imaginary part. The validation of dispersion relation with the classical case is made by using Debye asymptotic expansion which is the major highlight of this study. The width ratio and heterogeneity parameters of the layers, horizontal compressive initial stress and horizontal tensile initial stress are found to have a significant effect on the phase velocity and damped velocity of SH-waves. The obtained dispersion relation is found to be in well agreement with the classical Love-wave equation. Comparative study and graphical illustration has been made to reveal the some of the important facts.

2. Formulation and solution of the problem

In the present paper we consider two isotropic heterogeneous layers (M_1 and M_2) lying over an initially stressed isotropic homogeneous half-space (M_3) as shown in Fig. 1. Let us consider x-axis in the direction of wave propagation and along the common interface of medium M_2 and M_3. The z-axis of the rectangular co-ordinate system is pointing vertically downwards. The rigidity of the uppermost isotropic layer is a quadratic function of depth, whereas the density is constant. The rigidity and density of the sandwiched layer are varying exponentially with depth.

Now, for the propagation of SH-wave in x-direction and causing displacement only in the y-direction, we have the displacement components as

$$u = 0, \quad w = 0 \quad \text{and} \quad v = v(x, z, t)$$

Let us consider μ and ρ be the rigidity and density of the medium respectively. In the absence of body forces, the only non-vanishing equation of the motion [11] for the propagation of SH-wave is given by

$$\frac{\partial}{\partial x} p_{xy} + \frac{\partial}{\partial z} p_{yz} = \rho \frac{\partial^2 v}{\partial t^2}, \tag{2.1}$$

where

$$p_{xy} = \mu \frac{\partial v}{\partial x}$$

and

$$p_{yz} = \mu \frac{\partial v}{\partial z}.$$

For a homogeneous medium, where μ is constant i.e. independent of the space variables, the equation of motion for propagation of SH-wave wave in an isotropic homogeneous medium is given by

$$\nabla^2 v = \frac{1}{\beta^2} \frac{\partial^2 v}{\partial t^2}, \tag{2.2}$$

where

$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$$

and

$$\beta^2 = \frac{\mu}{\rho}.$$

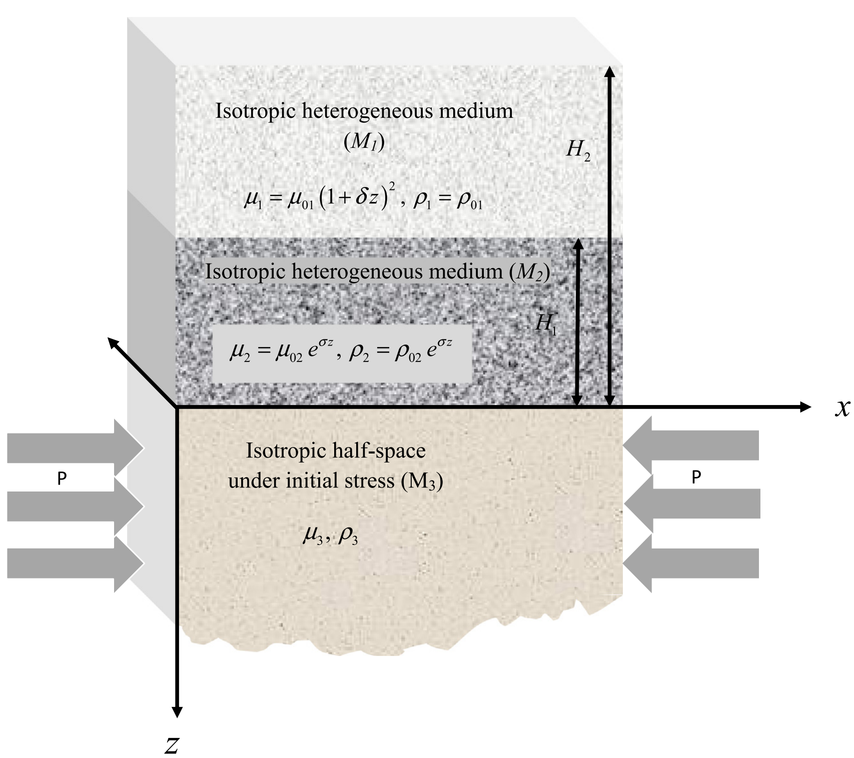

Fig. 1. Geometry of the problem.