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Response of circular flexible foundations subjected to horizontal and rocking motions

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ABSTRACT

A methodology using modal analysis is developed to evaluate dynamic vertical displacements of a circular flexible foundation resting on soil media subjected to horizontal and rocking motions. The influence of the soil reaction forces on the foundation is considered by introducing modal impedance functions, which can be determined by an efficient procedure with ring elements. The displacements of the foundation can then be easily solved by modal superposition. Parametric studies for modal responses of the flexible foundation indicate that the coupled response of the foundation is significantly influenced by relative stiffness among the foundation and the soil medium, vibration frequency range, foundation mass, and boundary contact conditions. The welded boundary condition should be considered to predict the coupling response while the relaxed boundary condition may be used to predict approximately the vertical displacements. As a foundation with a relative stiffness ratio more than three, it is found that the foundation can be considered as rigid to calculate coupling displacements. For a slightly flexible foundation, considerations of three modes are sufficient enough to obtain accurate foundation responses. Moreover, at low frequencies, the coupling effect due to higher mode can be neglected.

1. Introduction

Foundation vibration analysis plays a key role in solving soil–structure interaction problems, and has been investigated widely since 1930s. In the early stage, the foundation was often assumed as rigid and this assumption has been also considered in most of recent studies [1–4]. After 1970s, effects of the foundation flexibility are increasingly recognized and studied by many researchers [5–12]. In those studies, the considered soil conditions involve elastic half-space, layered soil, and poro-elastic medium; the considered dynamic loadings are vertical or rocking force; the interface between the foundation and the underlying soil medium is always assumed as frictionless.

The past studies have found that the rocking and horizontal vibrations of the rigid foundation are coupled. Triantafyllidis and Prange [13] studied the full coupling at the interface between the rigid foundation and soil medium. Not only rocking forces but also horizontal force can induce the vertical displacement. However, the coupled response of flexible foundations subjected to horizontal loadings is hardly investigated. Furthermore, flexible foundations are multiple-degree-of-freedom (MDOF) systems, dynamic behaviors of which are complicated and seriously affected by the position, the load distribution, and the foundation stiffness. Because the modal analysis is widely used in the engineering and features in reducing complexity of the MDOF system and amount of computational work, the modal method is suitable in this study. Recently, Chen and Hou [14] have investigated the vertical vibration of the flexible foundation by a modal method, and their results indicated that three modes are sufficient in the majority of analyses.

The primary objective of this paper is therefore to present a modal concept method to evaluate and analyze the dynamic vertical displacement of a flexible circular foundation laying on the soil medium subjected to harmonic horizontal forces and antisymmetric rocking forces, as shown in Fig. 1. Welded contact is considered between the foundation and the supporting soil. The foundation plate behaves according to the classical thin plate theory. In addition, the flexible behavior of the foundation is only considered in out of plane motion (i.e. vertical direction), and the in-plane rigidity of the foundation is assumed as rigid.

In this study, the modal equations of motion for a circular plate are firstly established. An efficient procedure is also introduced to calculate modal impedance functions to consider the interaction effect between the foundation and the underlying soil. The dynamic responses of the flexible foundation thus computed by the proposed method are then compared with existing solutions and numerical results by a computer program. Moreover, parametric studies are presented to evaluate the influence of primary parameters on the foundation responses.

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2. Formulation of interaction problem

2.1. Equations of motion

Considering a circular flexible foundation subjected to vertical and horizontal loads as shown in Fig. 1, the equations of harmonic motions with frequency ω in the z and x directions can be expressed, respectively, by the following equations [15]:

$$\sum F_z: \; D\nabla^4 w(r,\theta) - m_p\omega^2 w(r,\theta) = p_z(r,\theta) - q_z(r,\theta) \tag{1a}$$

$$\sum F_x: \; -\left(m_p \pi R^2\right)\omega^2\delta = P_x - Q_x \tag{1b}$$

where $\nabla^4 = \nabla^2\nabla^2$ and ∇^2 is the Laplacian operator. D is the flexural rigidity of the foundation plate and defined as follows:

$$D = \frac{E_p h_p^3}{12(1-v_p{}^2)} \tag{2}$$

where R, E_p, h_p, υ_p and m_p are radius, Young's modulus, thickness, Poisson's ratio and the mass per unit area of the foundation plate, respectively. w and δ are the vertical and horizontal displacements of the foundation plate, respectively; p_z and q_z are the vertical load per unit area of the foundation plate and the vertical contact stress, respectively; P_x and Q_x are the total horizontal load and the horizontal soil reaction, respectively; r and θ are the radial and circumferential direction, respectively.

In addition, the vertical load p_z is an antisymmetric rocking force (about the y-axis), and can be expressed as follows:

$$p_z(r,\theta) = \overline{p}_z(r)\cos(\theta) \tag{3}$$

where $\overline{p}_z(r)$ is the vertical load amplitude. Also, the vertical, radial, circumferential contact stresses (q_z, q_r, q_θ) and the horizontal soil reaction Q_x can be presented as follows:

$$\begin{Bmatrix} q_z(r,\theta) \\ q_\theta(r,\theta) \\ q_r(r,\theta) \end{Bmatrix} = \begin{Bmatrix} \overline{q}_z(r)\cos(\theta) \\ -\overline{q}_\theta(r)\sin(\theta) \\ \overline{q}_r(r)\cos(\theta) \end{Bmatrix} \quad \text{for} \quad 0 \le r \le R \tag{4}$$

$$Q_x = \pi\int_0^R r\left(\overline{q}_r(r) + \overline{q}_\theta(r)\right)dr \tag{5}$$

where $\overline{q}_z(r)$, $\overline{q}_r(r)$, and $\overline{q}_\theta(r)$ are the vertical, radial, and circumferential contact stress amplitudes, respectively.

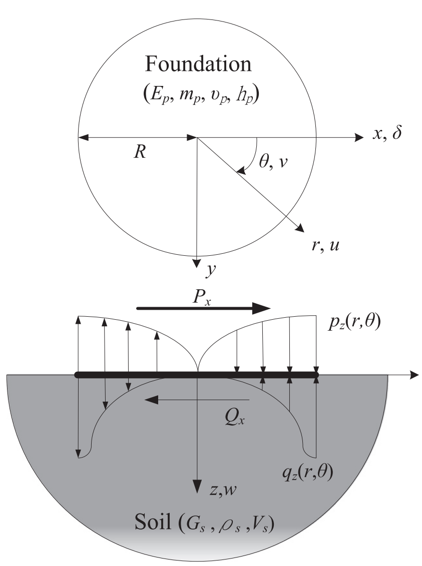

Fig. 1. A schematic of a flexible circular foundation resting on soil media.

2.2. Foundation deflection and contact stress in terms of modal displacements

The deflection of a circular foundation plate in z-direction can be represented by a series of free vibration modes for a circular plate as follows [5]:

$$w(r,\theta) = \sum_{i=0}^{N} Y_i \Phi_i(r,\theta) \tag{6a}$$

$$= \sum_{i=0}^{N} ||Y_i|| e^{i\theta_i}\Phi_i(r,\theta) \tag{6b}$$

where Y_i, $||Y_i||$, and θ_i are the modal displacement, the modal amplitude, and the modal phase angle, respectively; Φ_i (i=0, 1,..., N) denotes the mode shape of a circular plate for the ith mode.

Since the foundation behaves according to the classic thin plate theory and foundation deflections are antisymmetric about the y-axis, free vibration mode shapes of a circular foundation can be expressed as [15]

$$\Phi_i(r,\theta) = \overline{\varphi}_i(r)\cos(\theta) = \left(a_i J_1\left(\lambda_i\frac{r}{R}\right) + b_i I_1\left(\lambda_i\frac{r}{R}\right)\right)\cos(\theta) \tag{7}$$

where J_1 is the Bessel function of the first kind of order 1, I_1 is the modified Bessel function of the first kind of order 1. The values of a_i, b_i and λ_i determine the mode shape and are solved from the boundary conditions. For a surface foundation, the moments and shear forces at the edge of the foundation plate are zero, and the coefficients a_i, b_i and λ_i can be determined as follows [15]:

$$\frac{\lambda_i J_1(\lambda_i) - (1-\upsilon_p)J_2(\lambda_i)}{\lambda_i I_1(\lambda_i) - (1-\upsilon_p)I_2(\lambda_i)} = \frac{\lambda_i J_1(\lambda_i) - \lambda_i^2 J_2(\lambda_i) - (1-\upsilon_p)J_2(\lambda_i)}{\lambda_i I_1(\lambda_i) + \lambda_i^2 I_2(\lambda_i) - (1-\upsilon_p)I_2(\lambda_i)} \tag{8a}$$

$$\frac{b_i}{a_i} = \frac{\lambda_i J_1(\lambda_i) - (1-\upsilon_p)J_2(\lambda_i)}{\lambda_i I_1(\lambda_i) - (1-\upsilon_p)I_2(\lambda_i)} \tag{8b}$$

The modes are orthogonal and normalized as

$$\int_0^R \int_0^{2\pi} r\Phi_i(r,\theta)\Phi_j(r,\theta)dr\,d\theta = \begin{cases} \pi R^2 & i=j \\ 0 & i \ne j \end{cases} \tag{9}$$

Considering υ_p=0.25, the mode shapes are shown in Fig. 2. The mode shape Φ_i for $i>0$ denotes the flexible deformation in higher modes, and Φ_0 is the rigid body mode for rocking vibration and can be

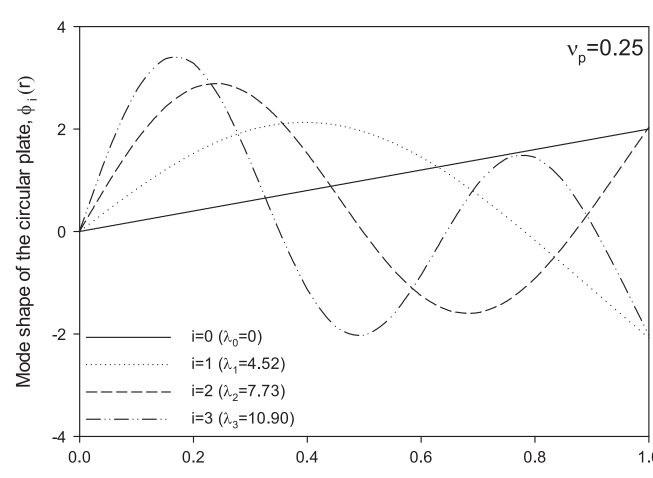

Fig. 2. Free vibration mode shapes of a circular plate for rocking vibration.