An optimization method for the generation of ground motions compatible with multi-damping design spectra

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Abstract

For some crucial engineering projects, a compatible time history of ground motions is required as a loading input for seismic dynamic analysis. The compatibility with multi-damping design spectra is critical to make sure the high reliability of analysis results. In previous methods, ground motions were hard to be tuned highly compatible with multi-damping design spectra. Hence we propose a method to optimize the compatibility of a seed ground motion history, which is either obtained from actual seismic records or generated from previous methods, by superimposing adjusted ground motions in the time domain through the $L_1$ norm minimization. The optimization process makes the influences of the superimposed ground motions to the response spectra be under control by constraining objective variables. The high precision of the method is demonstrated by a concrete example, in which the max difference between the response spectra of the optimized ground motions and the design spectra for RG1.60 data [1] is 5.06% under five different damping ratios, and all response spectra envelope the design spectra.

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1. Introduction

Seismic response history analysis (SRHA) is an approach to perform non-linear dynamic analysis for critical structures, such as dams and nuclear power plants [2,3]. In the procedure of SRHA, representative time histories are needed to perform the analysis, which may come from actual strong ground motion records or may be artificially generated.

The response spectrum of ground motions is defined as a plot of the peak value of a response quantity. The quantity, which may be a displacement, a velocity, or an acceleration, is a function of natural vibration period $T$ and damping ratio $\zeta$ of a linear single degree-of-freedom (SDOF) system. In this paper, the quantity is taken as the acceleration, and the terminologies time history and ground motions are interchangeable to represent the acceleration records, which are used as an input for structural analysis.

Seismic hazard is represented by the predicted ground motion intensity with rates of exceedance. For dynamic aseismic analysis of structures, ground motion with prescribed intensity levels is required and critical for the reliability of the analyzed results. The intensity levels could be peak ground acceleration (PGA) or spectral accelerations for a variety of periods, in which the latter consists of a target spectrum or called a design spectrum. For example, the conventional design spectra [1] and the Uniform Hazard Spectra (UHS) [4,5] are widely used. Compatibility with design spectra is a critical factor for the representability of time histories, and this has been prescribed in seismic design codes [1,6].

There are two main reasons to use spectrum compatible time histories. (1) The analysis result from spectrum compatible time histories has lower dispersion [7]. (2) Lots of regions in the world do not have sufficient ground motion records [8].

The structures, systems, and components of a nuclear power plant have different damping ratios. Thus it has been expected that the multi-damping response spectra of input ground motions should match the target design spectra well for performing the time history analysis. It is relatively easy to generate a spectrum compatible time history under a single damping ratio. However, under the requirements of multi-damping ratios, the max difference between response spectra and design spectra is hard to control [9].

The generation of ground motions compatible with design spectra has been an active research area over the past three decades. Earlier research works were reviewed by [10]. From the stochastic view of the time history, some works [11] represent the time history in the form of superimposed Fourier series, in which...
the Fourier amplitude and phase are adjusted iteratively to match the design spectra. To characterize the non-stationary behavior of the amplitude, various envelope functions are proposed [12–16].

Furthermore, it has been widely recognized that the frequency content in strong ground motion records is also non-stationary. In the work of [17,18], an adjustment framework has been proposed based on solving linear equations regarding to the weights of basis functions for the adjustment. Under this framework, evolutionary power spectra have been used to characterize the non-stationarity of frequency [19], and new kinds of wavelets are proposed which have better properties on stability and drift corrections on velocities and displacements [22–24]. Moreover, the directionality of ground motions is considered in the work of [25]. Besides wavelets for the adjustment, wavelet transform [26,27] is another sharp tool to make the time history be represented in the form of time-dependent frequency components. Recently, Hilbert–Huang transform is applied to retain the instantaneous frequency characteristics of ground motion records [28,29].

Besides the research to make the simulated ground motions approximate the characteristics of real records, another effort branch in this direction is to enhance the match precision between approximations. Refs. [17,18] extended the method of [11] through solving linear equations. However, when the method is used for many different frequencies and multi-damping ratios, the equations are over constrained and the algorithm may not converge in some cases. In recent works of RSPMatch [24], the convergence problem is addressed, but the fundamental problem of match precision is still unclear.

The main drawback of the previous methods on the ground motion adjustment is lacking the control of its influence to other responses besides peak responses. To overcome this problem, an optimization method is proposed to reduce the match error between response spectra of generated motions and target spectra in this study, in which the influence of the added ground motions on other responses is under control by the constraints in the optimization process. From the optimization view, each adjustment process is effective, and lots of characteristics of the original ground motions are retained through constraining objective variables.

2. The $L_{\infty}$ norm optimization

In this section, we introduce the main building block of our method, which is an algorithm to solve the problem of $L_{\infty}$ norm minimization. The algorithm will be employed in the adjustment procedure.

The $L_{\infty}$ norm of a vector $\mathbf{x}$ is the max absolute value of its components. In quasi-convex programming [33,34], the $L_{\infty}$ norm minimization problem is in the form of

$$\begin{align*}
opt & = \min_{\mathbf{x}} \gamma \\
\text{s.t.} & \quad g_1(\mathbf{x}) - \gamma g_2(\mathbf{x}) \leq 0 \\
& \quad \mathbf{x} \in \mathcal{X} \quad \gamma \geq 0.
\end{align*}$$

where $g_1(\mathbf{x}) - \gamma g_2(\mathbf{x}) \leq 0$ is a short form of $m$ inequalities $g_i(\mathbf{x}) - \gamma g_2(\mathbf{x}) \leq 0$, $i = 1, \ldots, m$. Also, $g_1(\mathbf{x})$ is convex and $g_2(\mathbf{x})$ is concave. Thus the feasible set $\Sigma = \{\mathbf{x} \mid g_1(\mathbf{x}) - \gamma g_2(\mathbf{x}) \leq 0\}$ is a convex set for fixed $\gamma$. Here the minimized $L_{\infty}$ norm of the vector $(g_{11}(\mathbf{x})/g_{21}(\mathbf{x}), \ldots, g_{1n}(\mathbf{x})/g_{2n}(\mathbf{x}))$ corresponds to the solution $\gamma$ of problem $O$.

For the classical bisection algorithm on $\gamma$, each iteration solves the following convex problem:

$$\begin{align*}
P_{\gamma} & : \text{Find } \mathbf{x} \\
\text{s.t.} & \quad g_1(\mathbf{x}) - \gamma g_2(\mathbf{x}) \leq 0 \\
& \quad \mathbf{x} \in \mathcal{X}.
\end{align*}$$

To solve the feasibility problem $P_{\gamma}$, we convert $P_{\gamma}$ into $Q_{\gamma}$

$$\begin{align*}
Q_{\gamma} & : \min_{\mathbf{x}, w} w \\
\text{s.t.} & \quad g_1(\mathbf{x}) - \gamma g_2(\mathbf{x}) \leq w \mathbf{1} \\
& \quad \mathbf{x} \in \mathcal{X}.
\end{align*}$$

Here $\mathbf{1} = (1, \ldots, 1)$ is a vector, of which all the components are equal to 1. The problem $Q_{\gamma}$ could be solved by primal–dual interior point methods [35]. Let us use $(\mathbf{x}^w, w^w)$ to represent the solution of the problem $Q_{\gamma}$, for a given $\gamma$. The problem $P_{\gamma}$ is feasible when $w^w \leq 0$, otherwise it is infeasible. Now we are ready to show the complete procedure in Algorithm 1.

Algorithm 1. Bisection algorithm.

**Input:** Initial interval $[l^1, u^1]$ subject to $l^1 \leq u^1 \leq u^1$

while true do

\(l_k \leftarrow (l_k + u_k)/2;\)

Solve $Q_{l_k}$ to get $\mathbf{x}^k, w$. 

if $w < 0$ then

\(\mathbf{x}^{k+1} \leftarrow \mathbf{x}^k;\)

\(u^{k+1} \leftarrow \max g_1(\mathbf{x}^k)/g_2(\mathbf{x}^k);\)

\(l^{k+1} \leftarrow l^k;\)

else

\(l^{k+1} \leftarrow l^k;\)

\(u^{k+1} \leftarrow u^k;\)

end

if $u^{k+1} - l^{k+1} < \epsilon$ then

\(\) return $(\mathbf{x}^k, u^{k+1});$

end

end

Although there are some more efficient and complex algorithms to solve the problem $O$ [34,36], we use the classical bisection algorithm for its easier implementation and stability.