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# Axisymmetric vibration of an elastic circular plate bonded on a transversely isotropic multilayered half-space

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ABSTRACT

Based on the analytical layer-element method, an analytical solution is proposed to determine the dynamic interaction between the elastic circular plate and transversely isotropic multilayered half-space. The dynamic response of the elastic circular plate is governed by the classical thin-plate theory with the assumption that the contact surface between the plate and soil is frictionless. The total stiffness matrix of the transversely isotropic multilayered half-space is acquired by assembling the analytical layer-element of each soil layer with the aid of the continuity conditions between adjacent layers. According to the displacement condition of coordination between the plate and soil, the dynamic interaction problem is reduced to that of multilayered transversely isotropic half-space subjected to axisymmetric harmonic vertical loading. Some numerical examples are given to study the vertical vibration of the plate, and the results indicate that the dynamic response of elastic circular plate depends strongly on the material properties of the soils, the rigidity of the plate, the frequency of excitation and the external load form.


## 1. Introduction

The problem of dynamic interaction between a loaded elastic plate and layered soils has caught significant attention due to the wide application of elastic circular foundations to various structures. The understanding of soil–structure interaction under the dynamic loading in civil engineering is of great importance to the design of machine foundation and dynamic hardness testing. On the other hand, the soils in natural state consist of different strata with varied properties due to the long-term sedimentation process. Therefore, the effect of the soil's inhomogeneity should be taken into consideration in the analysis of vertical vibration of plate–soils system to simulate real situation. In the past, various analytical and numerical methods have been employed to research dynamic response of rigid plate–layered soils system under vertical time-harmonic loads. In some paper, both of the rigid plate resting on the surface of an elastic half-space [1–3] or layered half-space [4,5] and the rigid circular plate buried at an arbitrary depth of an elastic half-space [6,7] or a poroelastic half-space [8,9] or a full-space [10] have been studied.

All studies mentioned above assume the circular plate to be rigid, however, to the author's point of view, the circular plate should better be considered as an elastic structure foundation in civil engineering since it acquires the foundation deflection and the dynamic contact pressure under the plate. In the aspect of the dynamic interaction of elastic circular plate with soils, some papers have been published in dealing with this problem by means of different foundation models. By introducing Fredholm integral equations of the second kind, the dynamic responses of the flexible circular plate on a viscoelastic medium subjected to time-harmonic vertical and rocking excitation are studied by Lin [11] and Iguchi and Luco [12]. Schmidt and Krenk [13] investigated the vibration of an elastic circular plate with an elastic half-space according to an integral equation method with a trigonometric expansion. Based on the stiffness matrix of multilayered soils and the finite difference energy method, Gucunski and Peek [14] analyzed the vertical oscillations of a flexible circular plate. Bo [15] combined Hankel and Abel transforms to take a research on the vertical vibration problem of elastic circular disc at the surface of poroelastic half-space. Senjuntichai and Sapsathiarn [16] obtained the vertical displacement of elastic circular plate on multilayered poroelastic medium by solving the equation of motion and the deflection equation of the circular plate. The static and dynamic responses of an elastic circular located on a two-parameter tensionless foundation are studied by Celep and Güler [17] by employing Galerkin's approximation technique. Based on

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the potential function for the wave equations in transversely isotropic half-space [18], Eskandari-Ghadi et al. [19] analyzed the tensionless–frictionless interaction of flexible annular foundation with a transversely isotropic multi-layered half-space. The researches mentioned above involve dynamic response of elastic plate attached to the surface of soils with various analytical methods and numerical techniques. A review of their study indicates that computational procedure on dynamic interaction between elastic circular plate and layered soils is relatively complicated.

This paper aims to present a new analytical method to simplify the computational procedure for achieving the vertical time-harmonic vibration response of the elastic circular disc on layered half-space based on the existing research results in Ref. [20]. According to the assumption of frictionless contact between the elastic plate and the surface of soils together with the continuity of interface conditions between the layers, the total stiffness matrix of soils which consists of the analytical layer-element of each soil layer can be established for acquiring the vertical dynamic displacement and contact stress of the soils. Due to the analytical layer-element without positive exponential functions, the proposed method in this paper is computationally efficient and stable. The accuracy of the presented method is proved by comparing with the published results. Finally, the influence of the soils properties and the plate stiffness on the response of the plate is investigated.

## 2. Description of the problem and its solution

### 2.1. Description of the problem

As shown in Fig. 1, an elastic circular plate bonded on a transversely isotropic multilayered half-space is set under the axisymmetric coordinate system. In view of the symmetry of the load and the soil system, a cylindrical coordinate system is adopted to suppress the angular dependence of the solution, where $z$-axil is normal to the surface of the soils and appears as the axis of symmetry of the soils. The time factor $e^{i\omega t}$ is henceforth suppressed from all expressions for brevity, and the body force of the medium is neglected in this paper. The elastic circular plate with radius $R$ is undergoing a prescribed axisymmetric time-harmonic vertical external loading $p(r,H_0)e^{i\omega t}$, with $p(r,H_0)$ and $\omega$ being the amplitude and circular frequency of the load, respectively, and $i=\sqrt{-1}$. In addition, $R_0$ and $R$ are the radii of the axisymmetric vertical external loading $p(r,H_0)e^{i\omega t}$ and the circular plate, respectively. The foundation is composed of different thicknesses of layered soils with properties of transversely isotropy. The parameters of the foundation in Fig. 1 are Young's modulus in the vertical direction $E_v$, Young's modulus in the horizontal direction $E_h$, shear modulus governing shear deformations of the planes of isotropy $G_v$, Poisson's ratios characterizing the lateral strain response in the plane of transverse isotropy to a stress acting parallelly and normally to the plane $\mu_h$ and $\mu_{vh}$, and the density of the soil layer $\rho$. The material parameters with subscript $j$ refer to the $j$th soil layer separately ($j=1,2,3\ldots n$; $n$ is determined by the number of natural layers and the calculation points) and the underlying half-space is the $(n+1)$th soil layer.

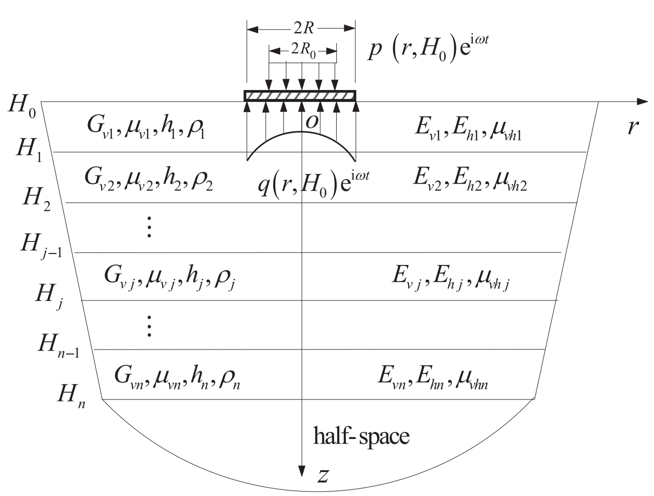


**Fig. 1.** A circular plate bonded on a multilayered half-space and subjected to axisymmetric harmonic external loading.

Based on the boundary conditions and the interaction relationship between the elastic circular plate and the soils, some measures are taken to obtain the solution of this interaction problem. With the assumption that the contact interface between the elastic circular plate and the soil is smooth, the interaction problem can be divided into two parts. The first part is that the elastic circular thin plate is subjected to an external loading $p(r,H_0)$ at the top of the circular plate and the ground reaction force $q(r,H_0)$ beneath the circular plate. Assuming that the elastic circular plate is attached to the surface of multilayered half-space, then the plate would never separate from the half-space. The second part is the transversely isotropic multilayered half-space with a loading $q(r,H_0)$ acting on the surface.

### 2.2. General solution for the elastic circular plate

Firstly, the solution for the elastic circular plate undergoing an external loading $p(r,H_0)$ at the top of the plate and the ground reaction force $q(r,H_0)$ underneath of the plate should be carried out. Based on the classical theory of elastic thin plate, the dynamic equation of the elastic thin plate can be expressed as follows:

$$D\nabla^4 w(r,H_0)-\rho_p h\omega^2 w(r,H_0)=p(r,H_0)-q(r,H_0) \tag{1}$$

where $D=E_p h^3/12(1-\mu_p^2)$ is the flexural rigidity of the plate; $\nabla^2=\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)$; $w$, $\rho_p$, $E_p$, $h$, $\mu_p$ are the deflection, the mass density, Young's modulus, thickness and Poisson's ratio of the circular thin plate, respectively. If the elastic circular plate is subjected to a axisymmetric harmonic uniform vertical patch loading $p_0$ with a radius of $R_0 (0 \le R_o \le R)$, $p(r,H_0)$ can be expressed as follows:

$$p(r,H_0)=\begin{cases} p_0 & 0<r\le R_0 \\ 0 & r>R_0 \end{cases} \tag{2a}$$

If the elastic circular plate is subjected to a vertical harmonic point load P acting at $r=0$, $p(r,H_0)$ can be expressed by

$$p(r,H_0)=\mathrm{P} \tag{2b}$$

For convenience in solving the partial differential equation mentioned above, the Hankel integral transform is employed to reduce the order of it. The $m$th-order Hankel integral transform of a function $f(r,H_j)$ with respect to the radial coordinate is defined as

$$\bar{f}(\xi,H_j)=\int_0^\infty f(r,H_j)\, rJ_m(\xi r)dr \tag{3a}$$

with its inversion formula:

$$f(r,H_j)=\int_0^\infty \bar{f}(\xi,H_j)\, \xi J_m(r\xi)d\xi \tag{3b}$$

where $J_m$ stands for the Bessel function of the first kind of order $m$; and $\xi$ denotes the Hankel transform parameter.

Applying the zero order Hankel integral transform with respect to the radial variable $r$, the governing Eq. (1) can be written as

$$D\xi^4\overline{w}(\xi,H_0)-\rho_p h\omega^2\overline{w}(\xi,H_0)=\overline{p}(\xi,H_0)-\overline{q}(\xi,H_0) \tag{4}$$