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Dynamic Reissner–Sagoci problem for a transversely isotropic half-space containing finite length cylindrical cavity

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ABSTRACT

A transversely isotropic linear elastic half-space containing a circular cylindrical cavity of finite length with a depth-wise axis of material symmetry is considered to be under the effect of a mono-harmonic torsional motion applied on a rigid circular disc with the same radius of the cavity and welded at the bottom of the cavity. With the aid of Fourier sine and cosine integral transforms, the mixed boundary value problem is reduced to a generalized Cauchy singular integral equation for the unknown shear stress. The Cauchy integral equation involved in this paper is analytically investigated such that the solution is written in the form of a known singular function multiplied by an unknown regular function. The regular part of the shear stress is numerically determined with the use of Gauss–Jacobi integration formula. Series representation of the stress and displacement are obtained, and it is shown that their degenerated form to the static problem of isotropic material is coincide with the existing solutions in the literature. To investigate the effects of material anisotropy and the length of cavity, the tangential displacement and the shear stress in between the rigid disc and the bottom of cavity are numerically evaluated and illustrated, where some differences are distinguished. With the differences illustrated in this paper for different length of cavity, it is recognized that the effect of length of cavity cannot be neglected in analysis and design. Different results for different degrees of anisotropy shows that the anisotropy of the material is a normal behavior, which should be considered in this kind of medium.

1. Introduction

The problem of determination of stresses and displacements in an elastic medium due to torsional static and dynamic rotation of a rigid disc in welded contact has been an interesting subject in both theoretical and practical mechanics. In this topic, evaluation of the stresses and displacements of an elastic half-space with a finite open cylindrical cavity is a considerable interest in both mathematical and engineering points of view. In the context of engineering, a study of this kind is relevant to foundation drilling, structural and mechanical designs, and borehole technology [10]. The first investigation in this class of problems is due to Westergaard [17], who studied an infinite isotropic solid containing an infinite cylindrical cavity with hydrostatic pressure acting on the wall of the cavity and presented some approximate results. A rigorous investigation of this problem was presented by Tranter [16] and Jordan [5], then treated the dynamic problem of a suddenly applied pressure over a finite interval of the infinite cavity. Because of the complexities encountered in the problem, numerical results were presented only at large distance away from the location of pressure. As a fundamental example of mixed boundary value problems in the theory of elastic wave propagation, the problem of forced torsional rotation of an elastic half-space has been first attracted by Reissner and Sagoci [14] and Sogoci [15]. That is why this class of problems is often referred to as Reissner–Sagoci problem. Some researchers, after Reissner and Sogoci, have investigated either the static or dynamic interaction of rigid disc and elastic isotropic half-spaces, among which Collins [2]; Williams [19]; and Pak and Sophores [9] are mentioned.

The attempt for the simulation of the mechanical features of different realistic materials in the analyses, however, requires expressions for anisotropic media. The form of anisotropy with the most common application is perhaps the case of transverse isotropy. Rajapakse and Gross [13], for example, have been interested in the influence of transverse isotropy on the axisymmetric response of a borehole. Rahimian et al. [12] have investigated the Reissner–Sagoci problem for transversely isotropic half-space.

The solutions of the generalized problem associated with a finite cylindrical cavity in a half-space would be of even greater both engineering and mathematical interest and challenge. It has

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Nomenclature

The following symbols are used in this paper:

a	radius of cylindrical cavity and rigid disc
C_s	shear wave velocity in the isotropic plane
$H_n^{(p)}$	Hankel function of first and second kinds ($p=1,2$) of order n
I_m	modified Bessel function of the first kind and mth order
i	$\sqrt{-1}$
J_m	Bessel function of the first kind and mth order
K_m	modified Bessel function of the second kind and mth order
l	depth of cylindrical cavity
$\hat{l}$	normalized depth of cylindrical cavity
$P_n^{(\gamma,\beta)}$	Jacobi polynomials
r	radial coordinate
$\hat{r}$	normalized radial coordinate
t	time variable
$u(r,z,t)$	displacement component in θ-direction
Y_m	Bessel function of the second kind and mth order
z	vertical coordinate
$\hat{z}$	normalized vertical coordinate
μ	shear modulus in the plane normal to the axis of symmetry
μ'	shear modulus in planes normal to the plane of transverse isotropy
Θ	rotation angle of the rigid disc about the z-axis
θ	angular coordinate
ρ	material density
τ_{ij} $(i,j=r,\theta,z)$	shear stress tensor
ω	angular frequency
ω_0	non-dimensional frequency
ξ	Fourier parameter

been found that the additional stiffness of the medium below the bottom of the hole can lead to a noticeable change of the response in the upper region. Pak and Abedzadeh [10] investigated rigorously the problem of torsional shear static traction acting on an open finite cylindrical cavity in an isotropic half-space in detail, and found the corresponding fundamental solution. They also extracted mathematically the resulting load-induced as well as shape-induced singularities in the response. Eskandari-Ghadi et al. [4] have extended this problem for both dynamic case and transversely isotropic material, where they have shown that the anisotropy cannot be neglected. Pak and Abedzadeh [11] has been interested in the static torsion of a rigid disc bonded to the bottom of a finite length cylindrical cavity exist in an elastic half-space, where they have used a combination of Fourier sine and cosine integral transforms to reduce the related mixed boundary value problem to a pair of integral equations, one of which possesses a generalized Cauchy singular kernel. They have applied the theory of analytic functions and Gauss–Jacobi integration formula to evaluate the solution of the mixed boundary value problem.

This paper is concerned with the elastostatic and elastodynamic forced torsion of a transversely isotropic linear elastic half-space containing a finite open circular cylindrical cavity. To attack the mixed boundary value problem, a transversely isotropic half-space containing a circular cylindrical cavity of finite length is considered as the domain of the problem in such a way that the material symmetry of the half-space is assumed to be both depth-wise and parallel to the axis of cylindrical cavity. A rigid disc welded on the bottom surface of the cavity is considered to be oscillatory moved with a mono-harmonic torsional motion. As Pak and Abedzadeh [11] did, both Fourier sine and cosine integral transforms are used to reduce the in hand mixed boundary value problem to a generalized Cauchy singular integral equation for the unknown shear stress. The Cauchy integral equation involved in this paper is analytically investigated and transformed to an equation, which is numerically solved, after which the shear stress, circumferential displacement, and the impedance function are determined in a straight manner. Excellent satisfaction of the boundary conditions, and the excellent agreement between the impedance function determined from this paper for the simpler case of surface rigid disc attached on an isotropic half-space and the existing results prove both the validity and accuracy of the solution reported in this paper. It is shown that neglecting either the degree of anisotropy or the length of cavity results in some wrong results, which means that none of these two parameters can be neglected.

2. Formulation of the problem

A transversely isotropic homogeneous linear elastic half-space containing circular cylindrical cavity with radius $a>0$ and depth $l \geq 0$ is considered in a cylindrical coordinate system (r,θ,z), with a depth-wise z-axis, in such a way that the material axis of symmetry of the medium is parallel to both the z-axis and the axis of cylindrical cavity. A rigid disc of radius a bounded on the medium at the bottom of the flat-ended cavity, as depicted in Fig. 1, is considered to be affected by a time-harmonic torsional rotation, $\Theta\, e^{i\omega t}$, with Θ and ω, respectively, being the angle and circular frequency of the motion. Because of axial symmetry of the boundary value problem, the displacement vector has only one non-vanishing component, i.e. $u_\theta = u(r,z,t)$. Following Pak and Abedzadeh [10,11] (see also Eskandari-Ghadi et al. [4]), it is convenient to define two different regions as indicated in Fig. 1 and find the response of each region with satisfying the boundary and continuity conditions. These two regions are defined as

$$Region\ 1 = \{(r,\theta,z) | r > a, 0 < \theta \leq 2\pi, z > 0\}, \tag{1}$$

$$Region\ 2 = \{(r,\theta,z) | 0 \leq r < a, 0 < \theta \leq 2\pi, z > l\}. \tag{2}$$

The second region defines an open cylindrical region under the cavity and the first region is the remaining part of the whole half-space, which is also an open region. In the absence of the body

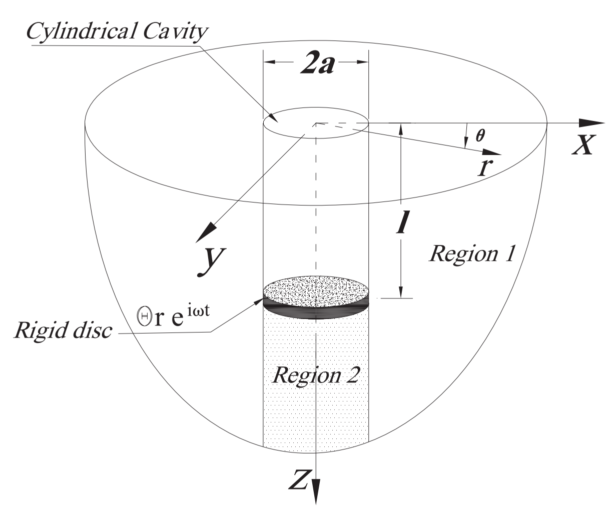

Fig. 1. A rigid disc on an indented transversely isotropic half-space.