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First-order doubly-asymptotic formulation of the direct stiffness method for elastodynamic problems

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ABSTRACT

A first-order formulation to analyze the dynamic response of layered soil profiles is presented as an alternative to the widely used second-order thin-layer method by the direct stiffness approach, including an efficient simulation of the underlaying elastic half-space. In contrast to the thin-layer method where response is expressed through a combination of second-order propagation modes, the proposed procedure uses first-order modal parameters that have the capacity to provide a good approximation in the complete wave number domain k, including the exact stiffness values for $k=0$ and $k\to\infty$, thus justifying its designation of doubly-asymptotic. This feature allows obtaining the exact soil profile response for static loads, while the proposed treatment of the elastic half-space reproduces naturally the radiation condition without a need of artificial damping. The capacity of the proposed formulation to solve elastodynamic problems is assessed by comparing its results with those of exact solutions available in the literature, and numerical solutions of rigid disks supported on the surface of different soil profiles.

1. Introduction

The direct stiffness method provides the basis for both the thin-layer method and for the new proposed method here, designated as first-order doubly-asymptotic formulation (FODAF). The exact stiffness matrices in the wave number domain for a finite stratum and for a half-space were given by Kausel and Roesset [1]. Calculation of the profile response by these methods is carried out for each frequency by transforming the excitation from the space domain to the wave number domain, calculating the displacements through the stiffness matrix of the soil profile and then applying the inverse transform into the space domain. The Hankel Transform allows calculating the response in cylindrical coordinates from the wave number domain to the spatial domain. The numerical implementations of this transform lead to inaccuracies due to singularities of the integrand that may be reduced by refining the discretization in the wave number k at an increased computational effort.

The thin-layer method (TLM) described in detail by Kausel [2] and Park [3] approximates the exact stiffness of a layer by the direct method through matrices that are independent of the wave number. The main advantage of this method lies in approximating the transcendental form of stiffness coefficients by algebraic expressions leading to a solution expressed in terms of eigenvalues associated with propagation modes of the soil profile. Such representation allows an analytical transformation into the space domain without additional loss of accuracy. In this approach the layers stiffness coefficients for wave numbers which tend to infinity are proportional to k^2 while the exact stiffness coefficients vary with k. This characteristic brings in the shortcoming that the method is not rigorously capable of representing static solutions of the soil profile. Representation of an underlying half-space in the thin-layer method is done by incorporating additional strata of increasing thickness up to a total thickness of 1.5 times the wave length for each frequency, and vertical and horizontal dashpots at the base of the lowest stratum as an approximation to the consistent boundary conditions as presented by Lysmer and Kuhlemeyer [4]. This last approach is only effective for plane and axisymmetric models according to Lin et al. [5], so that Oliveira Barbosa et al. [6] recently provided an improved approximation based on the perfectly matched layer technique (PML).

The first-order formulation proposed here relies on an expansion of the exact coefficients of the layer stiffness matrix up to the first power of k generating two independent matrices with respect to the wave number. In this way, the coefficients of the formulation are proportional to k for wave numbers tending to infinity as in the exact solution. The modal parameters result of the

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first-order in contrast to thin-layer method where they derive from second order matrices. Addition of auxiliary degrees of freedom (d.o.f.) to the stiffness matrices allows a good match with the exact coefficients in the wave number domain. These auxiliary d.o.f. are used to enlarge the modal model, and from the point of view of modal analysis can be considered as secondary or slave d.o.f., which once condensed to the primary or master d.o.f. allow to reproduce adequately the variations of the stiffness coefficients with respect to the wave number.

On the other hand, an experimental modal analysis technique is used to adjust the exact stiffness coefficients of the half-space through first-order modal parameters. The half-space modal model is then transformed into physical matrices that can be assembled with the matrices for the soil layers. Matching of the stiffness coefficients of the half-space is carried out both for real and imaginary components allowing a correct simulation of the radiation process and of the solution for the static cases. As a result, this formulation turns out to be doubly-asymptotic since it tends to the exact solution both when the wave number tends to zero and to infinity. Such feature is of interest in order to represent the soil profile response at low frequencies, including the static case, while retaining the advantage of the thin-layer method of the exact modal transformation from the wave number domain to the spacial domain. In addition, the foregoing formulation does not require artificial damping to avoid numerical problems. If required, material damping of the strata may be accounted for by adding it to the eigenvalues of the complete soil profile. Main issues related to the calculation of integrals of the Hankel Transform that arise in this formulation are discussed in the paper.

2. Direct stiffness method

Fig. 1 shows load and displacement components at the interfaces of the jth layer according to terminology adopted by Kausel and Roesset [1]. Cylindrical coordinates will be used here, although resulting matrices both for strata and for the half-space are also valid for plane wave fronts in cartesian coordinates.

Load vector P_j for each interface is transformed from the time domain t to the frequency domain ω through the Fourier Transform, azimutal coordinate θ is expressed in terms of Fourier series through integer numbers μ, and radial coordinate ρ is transformed to the wave number domain k by the Hankel Transform:

$$\overline{P}_j(k,\mu,\omega) = a_\mu \int_0^\infty \rho\, C_\mu \int_0^{2\pi} T_\mu \int_{-\infty}^{\infty} P_j(\rho,\theta,t)\, e^{-i\omega t} dt\, d\theta\, d\rho \qquad (1)$$

$$a_\mu = \begin{cases} 1/2\pi & \text{if} \quad \mu = 0 \\ 1/\pi & \text{if} \quad \mu \neq 0 \end{cases} \qquad (2)$$

$$C_\mu = \begin{bmatrix} \frac{d}{d(k\rho)} J_\mu(k\rho) & \frac{\mu}{k\rho} J_\mu(k\rho) & 0 \\ \frac{\mu}{k\rho} J_\mu(k\rho) & \frac{d}{d(k\rho)} J_\mu(k\rho) & 0 \\ 0 & 0 & -J_\mu(k\rho) \end{bmatrix} \qquad (3)$$

$$T_\mu = \begin{cases} \text{diag}\left[\cos(\mu\theta) \; -\sin(\mu\theta) \; \cos(\mu\theta)\right] : \text{symmetric loads respect to } x-\text{axis} \\ \text{diag}\left[\sin(\mu\theta) \; \cos(\mu\theta) \; \sin(\mu\theta)\right] : \text{anti}-\text{symmetric loads respect to } x-\text{axis} \end{cases} \qquad (4)$$

where $J_\mu(k\rho)$ is the Bessel function of μth order.

Force–displacement relations for a layer are expressed as:

$$K_j^{st} \overline{U}_j^{st} = \overline{P}_j^{st} \qquad (5)$$

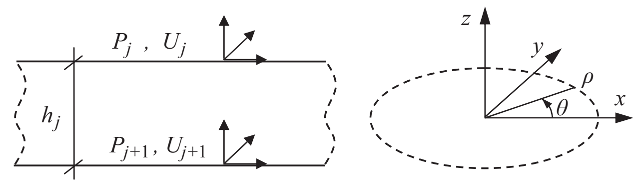

Fig. 1. Load and displacement components of the jth layer.

or:

$$\begin{bmatrix} K_{j,j} & K_{j,j+1} \\ K_{j+1,j} & K_{j+1,j+1} \end{bmatrix} \begin{bmatrix} \overline{U}_j \\ \overline{U}_{j+1} \end{bmatrix} = \begin{bmatrix} \overline{P}_j \\ \overline{P}_{j+1} \end{bmatrix} \qquad (6)$$

In cylindrical coordinates this last expression takes the form:

$$\begin{bmatrix} K_{11} & 0 & K_{13} & K_{14} & 0 & K_{16} \\ 0 & K_{22} & 0 & 0 & K_{25} & 0 \\ K_{13} & 0 & K_{33} & -K_{16} & 0 & K_{36} \\ K_{14} & 0 & -K_{16} & K_{11} & 0 & -K_{13} \\ 0 & K_{25} & 0 & 0 & K_{22} & 0 \\ K_{16} & 0 & K_{36} & -K_{13} & 0 & K_{33} \end{bmatrix} \begin{bmatrix} \overline{u}_\rho^j \\ \overline{u}_\theta^j \\ \overline{u}_z^j \\ \overline{u}_\rho^{j+1} \\ \overline{u}_\theta^{j+1} \\ \overline{u}_z^{j+1} \end{bmatrix} = \begin{bmatrix} \overline{\tau}_{\rho z}^j \\ \overline{\tau}_{\theta z}^j \\ \overline{\sigma}_z^j \\ \overline{\tau}_{\rho z}^{j+1} \\ \overline{\tau}_{\theta z}^{j+1} \\ \overline{\sigma}_z^{j+1} \end{bmatrix} \qquad (7)$$

where the degrees of freedom ρ and z (SV-P waves) are coupled, while d.o.f. θ (SH waves) is decoupled from the other ones.

The displacement vector for the complete profile is obtained as:

$$\overline{U} = K^{-1}\overline{P} = F\,\overline{P} \qquad (8)$$

where K represents the stiffness matrix of the profile obtained by assembling the individual layers and half-space matrices, while F represents the flexibility matrix of the soil profile.

The inverse transform to the space-time domain of the displacements obtained from Eq. (8) is carried out by:

$$U_j = \sum_{\mu=0}^{\infty} T_\mu \int_0^\infty k\, C_\mu \int_{-\infty}^{\infty} \overline{U}_j\, e^{i\omega t} d\omega\, dk \qquad (9)$$

The stiffness matrices in a non-dimensional form associated with the direct stiffness method are presented in what follows.

2.1. Layer stiffness matrices

The layer stiffness matrix for the d.o.f. associated with SV-P waves (Rayleigh modes) may be expressed as:

$$K_R^{st} = \omega \overline{\rho} V_S \overline{K}_R^{st} \qquad (10)$$

$$\overline{K}_R^{st} = \begin{bmatrix} \overline{K}_{11} & \overline{K}_{13} & \overline{K}_{14} & \overline{K}_{16} \\ \overline{K}_{13} & \overline{K}_{33} & -\overline{K}_{16} & \overline{K}_{36} \\ \overline{K}_{14} & -\overline{K}_{16} & \overline{K}_{11} & -\overline{K}_{13} \\ \overline{K}_{16} & \overline{K}_{36} & -\overline{K}_{13} & \overline{K}_{33} \end{bmatrix} \qquad (11)$$

$$\begin{aligned} \overline{K}_{11} &= \kappa(1-s^2)\frac{(T_s - rsT_r)}{Ds} \\ \overline{K}_{33} &= \kappa(1-s^2)\frac{(T_r - rsT_s)}{Dr} \\ \overline{K}_{14} &= \kappa(1-s^2)\frac{(rsT_rS_s - T_sS_r)}{Ds} \\ \overline{K}_{36} &= \kappa(1-s^2)\frac{(rsT_sS_r - T_rS_s)}{Dr} \\ \overline{K}_{13} &= \kappa(1-s^2)\frac{(1 - S_rS_s - rsT_rT_s)}{D} - \kappa(1+s^2) \\ \overline{K}_{16} &= \kappa(1-s^2)\frac{(S_r - S_s)}{D} \end{aligned} \qquad (12)$$

$$D = 2(S_rS_s - 1) + \left(\frac{1}{rs} + rs\right)T_rT_s \qquad (13)$$

$$\begin{aligned} T_r &= \tanh(r\kappa\eta) \quad T_s = \tanh(s\kappa\eta) \\ S_r &= \text{sech}(r\kappa\eta) \quad S_s = \text{sech}(s\kappa\eta) \end{aligned} \qquad (14)$$