Soil Dynamics and Earthquake Engineering 66 (2014) 415–428

Contents lists available at ScienceDirect

Soil Dynamics and Earthquake Engineering

journal homepage: www.elsevier.com/locate/soildyn

Approximate soil–structure interaction analysis by a perturbation approach: The case of soft soils

Armando Lanzi [a],*, J. Enrique Luco [b]

[a] Sapienza Università di Roma, Rome, Italy and Department of Structural Engineering, University of California, San Diego, USA
[b] Department of Structural Engineering, University of California, San Diego, La Jolla, CA 92093-0085, USA

ARTICLE INFO

Article history:
Received 19 May 2013
Received in revised form
12 December 2013
Accepted 8 March 2014

Keywords:
Soil–structure interaction
Dynamics
Perturbation
Modal analysis
Seismic

ABSTRACT

An approximate solution of the classical eigenvalue problem governing the vibrations of a relatively stiff structure on a soft elastic soil is derived through the application of a perturbation analysis. The full solution is obtained as the sum of the solution for an unconstrained elastic structure and small perturbing terms related to the ratio of the stiffness of the soil to that of the superstructure. The procedure leads to approximate analytical expressions for the system frequencies, modal damping ratios and participation factors for all system modes that generalize those presented earlier for the case of stiff soils. The resulting approximate expressions for the system modal properties are validated by comparison with the corresponding quantities obtained by numerical solution of the eigenvalue problem for a nine-story building. The accuracy of the proposed approach and of the classical normal mode approach is assessed through comparison with the exact frequency response of the test structure.

1. Introduction

In a companion paper [1], the authors utilized a perturbation approach to study the linear dynamic soil–structure interaction problem for an elastic structure supported on a relatively stiff soil. The analysis resulted in new approximate analytical expressions for the system frequencies, modal damping ratios and participation factors for all system modes which generalized those presented earlier by Bielak [2–3], Jennings and Bielak [4] and Veletsos and Meek [5] for the fundamental mode of a soil–structure system. The purpose of the present paper is to consider the other extreme limiting case corresponding to a structure supported on a relatively soft soil. Again, the objective is to present new approximate analytical expressions for the system frequencies, modal damping ratios and participation factors for all system modes for this case. These expressions would generalize those presented by Beredugo and Novak [6] for the case of a rigid structure on a flexible soil.

The use of the perturbation approach is selected as an alternative to a purely numerical, approximate modal analysis [7–13] of the linear dynamic soil–structure interaction problem, or to the use of the more accurate Foss' method [2,4,14], or a solution in the frequency domain. The approach has the advantage of leading to analytical expressions which offer considerable physical insight into the nature of the soil–structure interaction effects. However, the perturbation approach does not cure the basic limitations of the approximate modal analysis for soil—structure interaction problems described, among others, by Thomson et al. [15], Clough and Mojtahedi [16], Warburton and Soni [17], and Vaidya et al. [18].

2. Statement of the problem and classical modal approach

Consider the problem of forced vibrations of a linear elastic structure resting on a rigid foundation supported on a viscoelastic soil. The system is excited by elastic waves propagating through the soil and/or by external forces. The superstructure is discretized into a set of L nodal masses interconnected by massless elastic members; the rigid foundation may be partially embedded in the soil; and the soil is represented by a continuous,three-dimensional elastic or viscoelastic half-space. The fundamental equations of motion of the soil–structure system as well as those pertaining to the approximate classical modal approach have been presented in detail in the companion paper for stiff soils [1]. Here, we briefly recall the principal equations in order to facilitate the derivations that will follow.

The deformed configuration of the superstructure is described in terms of a $N \times 1$ $(N = 6L)$ vector $\{U_b\} = (\{u_1\}^T, \{u_2\}^T, ..., \{u_L\}^T)^T$ of generalized relative displacements of the nodes $\{u_i\} = (\Delta_{ix}, \Delta_{iy}, \Delta_{iz}, \theta_{ix}, \theta_{iy}, \theta_{iz})^T$ with respect to a frame of reference attached to the moving rigid foundation. The generalized total displacement

* Corresponding author.
E-mail addresses: alanzi@ucsd.edu, armando.lanzi@uniroma1.it (A. Lanzi).

http://dx.doi.org/10.1016/j.soildyn.2014.08.001
0267-7261/

vector for the superstructure $\{U_{bt}\}$, which describes the motion of the nodes with respect to a fixed frame of reference, is given by

$$\{U_{bt}\} = [\alpha]\{U_o\} + \{U_b\} \tag{1}$$

where $\{U_o\} = (\Delta_{ox}, \Delta_{oy}, \Delta_{oz}, \theta_{ox}, \theta_{oy}, \theta_{oz})^T$ is the total foundation motion at a point of reference in the foundation and $[\alpha]$ is a $N \times 6$ rigid-displacement influence matrix. The total foundation motion $\{U_o\}$ is given by

$$\{U_o\} = \{U_o^*\} + \{U_s\} \tag{2}$$

where $\{U_o^*\}$ is the foundation input motion, and $\{U_s\} = (\Delta_{sx}, \Delta_{sy}, \Delta_{sz}, \theta_{sx}, \theta_{sy}, \theta_{sz})^T$ is the relative motion of the foundation with respect to the input motion. The foundation input motion includes the effect of scattering of the seismic waves by the foundation [19].

For harmonic excitation, the motion of the superstructure and of the foundation is governed by the following system of equations (e.g., Lee and Wesley [20]; Luco [21])

$$\begin{bmatrix} M_b & M_b\alpha \\ \alpha^T M_b & M_{oo} \end{bmatrix} \begin{Bmatrix} \ddot{U}_b \\ \ddot{U}_s \end{Bmatrix} + \begin{bmatrix} C_b & 0 \\ 0 & C_s \end{bmatrix} \begin{Bmatrix} \dot{U}_b \\ \dot{U}_s \end{Bmatrix} + \begin{bmatrix} K_b & 0 \\ 0 & K_s \end{bmatrix} \begin{Bmatrix} U_b \\ U_s \end{Bmatrix} = \begin{Bmatrix} F_b \\ F_o \end{Bmatrix} - \begin{bmatrix} M_b\alpha \\ M_{oo} \end{bmatrix} \{\ddot{U}_o^*\} \tag{3}$$

where $[M_b], [C_b], [K_b]$ are the mass, damping and stiffness matrix for the superstructure on a fixed base, $[M_o]$ is the mass matrix of the foundation, $[M_{oo}] = [M_o] + [\alpha]^T[M_b][\alpha]$, $[K_s(\omega)] + i\omega[C_s(\omega)]$ represents the foundation impedance matrix, and $\{F_b\}$, $\{F_o\}$ are the generalized external forces acting on the superstructure and foundation, respectively. In Eq. (3), the harmonic time depending factor $e^{i\omega t}$ is omitted for brevity, and the displacement vectors $\{U_b\}$ and $\{U_s\}$ are frequency-dependent.

Let $\{\tilde{\phi}_j\}$ be the jth mode and $\tilde{\omega}_j$ the corresponding natural frequency of the un-damped building-foundation system. These quantities satisfy the eigenvalue problem

$$\begin{bmatrix} K_b & 0 \\ 0 & K_s(\tilde{\omega}_1) \end{bmatrix} \{\tilde{\phi}_j\} = \tilde{\omega}_j^2 \begin{bmatrix} M_b & M_b\alpha \\ \alpha^T M_b & M_{oo} \end{bmatrix} \{\tilde{\phi}_j\} \tag{4}$$

in which the frequency-dependent stiffness matrix is approximated by a constant value corresponding to the fundamental frequency of the system. Iterations are required to evaluate $\tilde{\omega}_1$ and obtain the corresponding constant stiffness matrix. Assuming as an approximation that the system admits decomposition into classical normal modes, the $(6L+6)$ vector of generalized relative displacements $\{U\} = (\{U_b\}^T, \{U_s\}^T)^T$ is written as

$$\{U\} = [\tilde{\Phi}]\{\tilde{\eta}\} = \sum_{j=1}^{N+6} \{\tilde{\phi}_j\}\tilde{\eta}_j \tag{5}$$

where $[\tilde{\Phi}]$ is the modal matrix and $\{\tilde{\eta}\}$ is the vector of modal amplitudes. Following the standard procedure, and neglecting the off-diagonal terms of the reduced modal damping matrix $[\tilde{\Phi}]^T[C][\tilde{\Phi}]$, Eqs. (3) and (5) lead to the system of $N+6$ uncoupled equations

$$\ddot{\tilde{\eta}}_j + 2\tilde{\omega}_j\tilde{\xi}_j\dot{\tilde{\eta}}_j + \tilde{\omega}_j^2\tilde{\eta}_j = \{\tilde{\phi}_j\}^T \begin{Bmatrix} F_b \\ F_o \end{Bmatrix} - \left\{\tilde{\beta}_j\right\}\left\{\ddot{U}_o^*\right\}, \quad j = 1,\ N+6 \tag{6}$$

The modal damping ratios and participation factors for seismic excitation are defined, as usual, by

$$2\tilde{\xi}_i\tilde{\omega}_i\tilde{M}_i = \{\tilde{\phi}_{bi}\}^T[C_b]\{\tilde{\phi}_{bi}\} + \{\tilde{\phi}_{si}\}^T[C_s]\{\tilde{\phi}_{si}\} \tag{7a}$$

$$\{\tilde{\beta}_i\}^T = \frac{1}{\tilde{M}_i}(\{\tilde{\phi}_{bi}\}^T[M_b][\alpha] + \{\tilde{\phi}_{si}\}^T[M_{oo}]) \tag{7b}$$

where $[C_b]$ is the damping matrix of the superstructure (assumed to admit classical damping) and $[C_s]$ is the damping matrix of the foundation, i.e. the imaginary part of the impedance matrix divided by ω. It is worth noting that, when computing the modal damping ratios, the damping matrix of the system is evaluated at each system natural frequency, in an effort to consider in an approximate fashion the frequency dependence of the imaginary part of the impedance coefficients.

3. Perturbation approach for stiff structures on soft soils

The perturbation approach for a flexible structure supported on a soft soil starts by considering that the foundation stiffness matrix $[K_s]$ is proportional to a characteristic soil shear modulus $G = \rho\beta^2$, where ρ is a characteristic soil density and β is a characteristic shear wave velocity in the soil. Following the same approach adopted in [1], it is possible to define a dimensionless quantity $(\beta/\omega_1 a)$, where a is a characteristic dimension of the foundation and ω_1 is the fundamental fixed-base frequency of the superstructure, which quantifies the relative stiffness between the structure and the foundation soil. In the case of a relatively stiff structure supported by a flexible soil, the dimensionless parameter $\varepsilon = (\beta/\omega_1 a)^2$ is small and the full solution can be seen as the sum of the solution for a perfectly-flexible soil and small perturbing terms, which can be expressed in a power series with respect to the small parameter ε. To start, the stiffness matrix of the foundation is written as

$$[K_s] = \varepsilon[\overline{K}_s] \tag{8}$$

where $[\overline{K}_s]$ represents the stiffness matrix of the foundation normalized by a term proportional to the square of the soil shear wave velocity.

The eigenvalues $\tilde{\lambda} = \tilde{\omega}^2$ and the displacements of interest can be expanded in terms of series of ε

$$\begin{aligned} \tilde{\lambda} &= \tilde{\lambda}_0 + \varepsilon\tilde{\lambda}_1 + \varepsilon^2\tilde{\lambda}_2 + O(\varepsilon^3) \\ \{U_b\} &= \{U_{b0}\} + \varepsilon\{U_{b1}\} + \varepsilon^2\{U_{b2}\} + O(\varepsilon^3) \\ \{U_s\} &= \{U_{s0}\} + \varepsilon\{U_{s1}\} + \varepsilon^2\{U_{s2}\} + O(\varepsilon^3) \end{aligned} \tag{9}$$

To obtain expressions for the coefficients of the series $\tilde{\lambda}_0, \tilde{\lambda}_1, \tilde{\lambda}_2, U_{b0}, U_{b1}, U_{b2}$ and U_{s0}, U_{s1}, U_{s2} it is necessary to substitute Eqs. (8) and (9) into Eq. (4), and collect and set to zero the terms multiplying $\varepsilon^0, \varepsilon^1$ and ε^2. The approach leads to the following equations for the zero-, first- and second-order terms

$$\left(\begin{bmatrix} K_b & 0 \\ 0 & 0 \end{bmatrix} - \tilde{\lambda}_0 \begin{bmatrix} M_b & M_b\alpha \\ \alpha^T M_b & M_{oo} \end{bmatrix}\right) \begin{Bmatrix} U_{b0} \\ U_{s0} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{10}$$

$$\left(\begin{bmatrix} K_b & 0 \\ 0 & 0 \end{bmatrix} - \tilde{\lambda}_0 \begin{bmatrix} M_b & M_b\alpha \\ \alpha^T M_b & M_{oo} \end{bmatrix}\right) \begin{Bmatrix} U_{b1} \\ U_{s1} \end{Bmatrix} = \begin{bmatrix} \tilde{\lambda}_1 M_b & \tilde{\lambda}_1 M_b\alpha \\ \tilde{\lambda}_1\alpha^T M_b & \tilde{\lambda}_1 M_{oo} - \overline{K}_s \end{bmatrix} \begin{Bmatrix} U_{b0} \\ U_{s0} \end{Bmatrix} \tag{11}$$

$$\left(\begin{bmatrix} K_b & 0 \\ 0 & 0 \end{bmatrix} - \tilde{\lambda}_0 \begin{bmatrix} M_b & M_b\alpha \\ \alpha^T M_b & M_{oo} \end{bmatrix}\right) \begin{Bmatrix} U_{b2} \\ U_{s2} \end{Bmatrix} = \begin{bmatrix} \tilde{\lambda}_1 M_b & \tilde{\lambda}_1 M_b\alpha \\ \tilde{\lambda}_1\alpha^T M_b & \tilde{\lambda}_1 M_{oo} - \overline{K}_s \end{bmatrix} \begin{Bmatrix} U_{b1} \\ U_{s1} \end{Bmatrix} + \tilde{\lambda}_2 \begin{bmatrix} M_b & M_b\alpha \\ \alpha^T M_b & M_{oo} \end{bmatrix} \begin{Bmatrix} U_{b0} \\ U_{s0} \end{Bmatrix} \tag{12}$$

from which $\tilde{\lambda}_0, U_{b0}, U_{s0}, \tilde{\lambda}_1, U_{b1}, U_{s1}, \tilde{\lambda}_2, U_{b2}, U_{s2}$ can be obtained in sequence. In addition, to determine some of these quantities, the system modal mass $\tilde{M}_i$ and the system modal stiffness $\tilde{K}_i$ given by

$$\tilde{M}_i = \left(U_b^T,\ U_s^T\right) \begin{bmatrix} M_b & M_b\alpha \\ \alpha^T M_b & M_{oo} \end{bmatrix} \begin{Bmatrix} U_b \\ U_s \end{Bmatrix} \tag{13a}$$

$$\tilde{K}_i = U_b^T K_b U_b + U_s^T K_s U_s \tag{13b}$$

need to be considered.