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Stochastic dynamic stiffness of a surface footing for offshore wind turbines: Implementing a subset simulation method to estimate rare events

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ABSTRACT

The purpose of this study, which concerns the stochastic dynamic stiffness of foundations for large offshore wind turbines, is to quantify uncertainties related to the first natural frequency of a turbine supported by a surface footing and to estimate the low event probabilities. Herein, a simple model of a wind turbine structure with equivalent coupled springs at the base is calibrated with the mean soil property values. A semianalytical solution, based on the Green's function for a layered half-space is utilized for estimation of foundation responses. Soil elastic modulus and layer depth are considered as random variables with lognormal distributions. The uncertainties are quantified, and the estimation of rare events of the first natural frequency is discussed through an advanced reliability approach based on subset simulation. This analysis represents a first step in the estimation of the safety with respect to the failure of a turbine in the fatigue limit state.

1. Introduction

To avoid collapse of the foundation and structure in the ultimate limit state and unacceptably large displacements in the serviceability limit state, offshore wind turbine foundations are designed in accordance with standards. Current standards are based on partial factors of safety or total safeties and assume quasi-static loading. Of special concern during design is the fatigue life, which may be significantly reduced due to cyclic loading from the wind and waves. In this context, a reliable estimate of the first natural frequency of the combined foundation and turbine structure is of major importance. Typically, the first natural frequency will be close to the frequency of the blades passing the turbine tower and only slightly higher than the excitation frequency from the waves. To avoid dynamic amplification of the response, the first natural frequency of the wind turbine structure, including its foundation, must lie within a narrow range. Unfortunately, the uncertainty of the natural frequencies cannot be quantified by a deterministic method based on partial factors of safety or total safeties.

Several studies have modeled the foundation stiffness through probabilistic approaches. Pinnaduwa and Lacasse [1] suggested a probabilistic procedure to estimate the equivalent linear soil spring stiffness of a foundation for gravity platforms on clay loaded under undrained conditions. Manohar and Adhikari developed a finite element (FE)-based methodology to determine the dynamic stiffness matrix of Euler–Bernoulli beams with randomly varying flexural and axial rigidity, mass density, and foundation elastic modulus values [2]. Using a nonparametric method, Cottereau et al. [3] constructed a probabilistic model for impedance matrices. Andersen et al. [4] studied a laterally loaded monopile for an offshore wind turbine foundation. They generated a probabilistic model to obtain the distribution of the natural frequency of a wind turbine and assumed a random process for the undrained shear strength of a clay soil. They also used an asymptotic sampling method as an advanced reliability estimation approach.

Using a reliability-based design, the present study examines the probability distributions of the natural frequency and the surface footing stiffness for an offshore wind turbine. The results indicate that these distributions are affected by the underlying soil uncertainties. In this regard, a semianalytical solution model is utilized to estimate the foundation stiffness and the first natural frequency of a wind turbine. A Monte Carlo process is used to conduct a probabilistic simulation. At the next step, an efficient probabilistic method is applied to estimate low probabilities at the tails of the distributions.

The dynamic response of footings has been studied by several researchers with various proposed models. Luco and Westmann [5] and Veletsos and Nair [6], who studied the influence of material

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damping, considered torsional vibrations of a rigid circular footing on a homogeneous half-space. Liou and Chung [7] developed a numerical scheme for the calculation of torsional, vertical, horizontal, coupling, and rocking impedances in the frequency domain for axial-symmetric foundations embedded in layered media. To analyze rigid circular foundations in a saturated porous layered half-space media, Lee et al. [8] developed a half-space FE and a consistent transmitting boundary in a cylindrical coordinate system. Novak and Sachs [9] and Avilés and Pérez-Rocha [10] reported approximate closed-form solutions for the torsional impedance of circular embedded foundations. Krenk and Schmidt [11] presented a closed-form solution for the vertical impedance of a flexible circular foundation on a homogeneous half-space, whereas Yong et al. [12] considered a circular footing on a layered soil. Veletsos and Wei [13] examined the rocking and horizontal sliding of circular footings on a homogeneous half-space, and Luco [14] presented a solution for a rigid circular footing on the surface of a stratum.

The present study utilizes a semianalytical solution, based on the Green's function for a layered half-space and a discretized traction distribution at the soil–foundation interface [15]. The soil is assumed to be homogeneous and the uncertainty related to the elastic modulus is assumed modeled by a log-normal distribution. In the same manner, the layer depth is introduced as a series of stochastic variables with a log-normal distribution. A Monte Carlo simulation (MCS) is carried out for numerous realizations of the random variables. In this manner, the distribution of the foundation stiffnesses and the first natural frequency of the wind turbine are numerically determined. The results show that many simulations are required to obtain a fine and smooth curve for the tail of the distribution, where low-probability events occur. This observation indicates that an advanced stochastic method would be helpful, particularly in cases of expensive simulations.

Many methods have been developed to estimate low-probability events and to perform reliability analyses. The most famous methods include first- and second-order reliability methods, the response surface method, and simulation approaches (also known as Monte Carlo methods). Au and Beck [16] developed a controlled Monte Carlo method, called the subset simulation (SS) method, which is abundantly used due to its efficiency [17–22]. Katafygiotis and Cheung [17] designed a benchmark study to evaluate the efficiency of reliability methods for large systems, focusing on the performance of the spherical SS and auxiliary domain methods. In another study, Katafygiotis and Cheung [18] evaluated the reliability of inelastic structural systems subjected to Gaussian random excitations by a modified SS procedure. Au et al. [19] presented three versions of the SS approach, which they used to perform a reliability analysis of three benchmark problems. They showed that SS methods are efficient for high-dimensional problems. Santoso et al. [20] proposed a modified Metropolis–Hastings (M–H) algorithm with reduced chain correlation, which they found to be useful for reliability analysis using the SS method. The regenerative adaptive SS method is a modification of the SS approach that was generated from a comparison with several sample generator algorithms [22].

In this paper, the SS method is used to estimate low-probability events in the fine and smooth curve at the distribution tails. The influence of the uncertainty of each soil property is addressed through a sensitivity analysis, by using the most significant uncertain parameters (herein, the elastic modulus and layer depth) as random variables. As a main conclusion, estimations of the distributions by the SS are more efficient than estimations by MCS when low probabilities are targeted (*e.g.*, $P<10^{-2}$). The results reveal a high probability that the natural frequency may lie in the resonance domain. Furthermore, log-normal behavior can be assigned to the stiffness distributions when the random variables are log-normally distributed. Although the proposed simple model cannot properly represent the real behavior of a wind turbine system, the results indicate that advanced numerical solutions (*e.g.*, advanced MCS) are good approaches for the reliability-based design of wind turbine foundations for cases in which an analytical solution is not available, such as for a highly nonlinear case with many degrees of freedom.

2. Work contribution and problem definition

Knowledge of the dynamic responses of a wind turbine is particularly important for the fatigue-based design process, in which the first natural frequency performs a pivotal role. The coincidence of the first natural frequency with the wind turbine-contributed dynamic force frequencies may lead to large amplitude stresses, accelerating fatigue. To avoid resonance, several design strategies may be considered, based on mass imbalances ($1P$), blade passing ($3P$), and high frequencies of the sea state that are generally close to the first natural frequency of the wind turbine.

Fig. 1 shows design cases according to frequency ranges for wind turbines with a variable rotor rate [4]. This figure illustrates the resonance diagram that is normally used to visualize the fundamental frequencies and design ranges. Any design strategy seeks to achieve a tower and foundation that contributes safe natural frequencies. Each strategy has certain advantages and disadvantages, depending on the types and dimensions of the

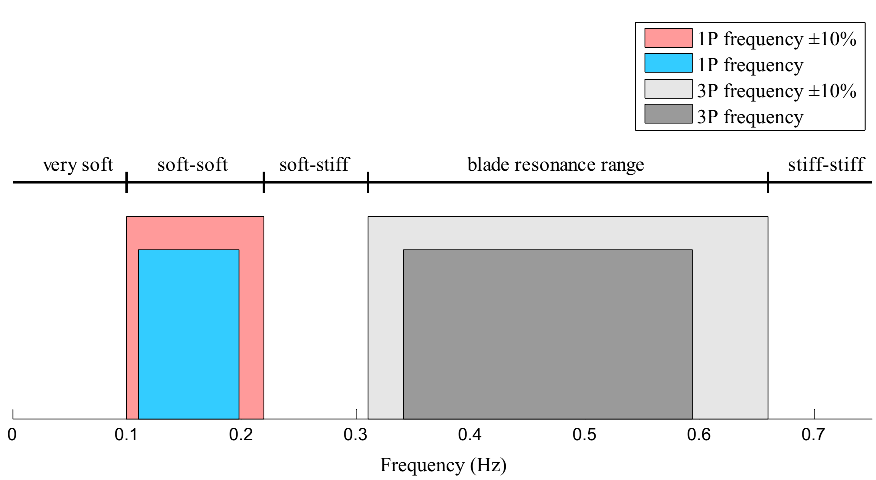

Fig. 1. Design strategies for the wind turbines by Andersen et al. [4], after [23].