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Numerical modelling of wave propagation in anisotropic soil using a displacement unit-impulse-response-based formulation of the scaled boundary finite element method

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ABSTRACT

An efficient method for modelling the propagation of elastic waves in unbounded domains is developed. It is applicable to soil–structure interaction problems involving scalar and vector waves, unbounded domains of arbitrary geometry and anisotropic soil. The scaled boundary finite element method is employed to derive a novel equation for the displacement unit-impulse response matrix on the soil–structure interface. The proposed method is based on a piecewise linear approximation of the first derivative of the displacement unit-impulse response matrix and on the introduction of an extrapolation parameter in order to improve the numerical stability. In combination, these two ideas allow for the choice of significantly larger time steps compared to conventional methods, and thus lead to increased efficiency. As the displacement unit-impulse response approaches zero, the convolution integral representing the force–displacement relationship can be truncated. After the truncation the computational effort only increases linearly with time. Thus, a considerable reduction of computational effort is achieved in a time domain analysis. Numerical examples demonstrate the accuracy and high efficiency of the new method for two-dimensional soil–structure interaction problems.

1. Introduction

Dynamic soil–structure interaction is of crucial importance in a number of engineering applications, such as the design of long-span bridges or high-rise buildings in areas prone to earthquake and wind loading. Taking into account of the soil–structure interaction in a structural analysis poses a number of challenges. Typically, the soil covers a very large domain and is thus idealised as an unbounded medium. The greatest difficulty in the dynamic analysis of an unbounded domain is to satisfy the radiation condition. In addition, most soils are not isotropic, and the correct modelling of the anisotropic soil in soil–structure interaction analyses is challenging. To consider the material and geometrical non-linearity occurring in the structure, it is highly desirable to model the wave propagation directly in the time domain.

Over the last forty years, a large number of numerical methods for the dynamic analysis of unbounded domains have been developed. This is reflected in many review articles [1–6]. Most existing approaches, such as the boundary element method [7,8] and the thin-layer method [9–11] can be classified as either rigorous or approximate. In general, rigorous methods are global in time and space and thus computationally expensive. In the boundary element method, the governing differential equations in the unbounded domain are expressed as boundary integral equations using a fundamental solution, which can be difficult to obtain for general anisotropic materials. The thin-layer method is a semi-analytical technique for horizontally layered media, which is formulated in the frequency domain. Anisotropy has been addressed in Refs. [9,10].

In approximate methods, the response at a specific location is approximately evaluated from the excitation during a limited past time (temporally local) and at its nearby region (spatially local). Such specific locations are referred to as artificial boundaries, which have to be located sufficiently far away from the domain of interest, in order to obtain results of acceptable accuracy. Classical low-order boundary conditions as well as first high-order artificial boundaries have been summarised in [4,12]. Here, terms higher than the second order result in complex formulations. Instability may also occur [13]. Numerous approaches to overcome these problems by introducing auxiliary variables have been developed. A summary of these methods can be found in Ref. [14]. The extension of these artificial boundaries to elastic waves in anisotropic unbounded domains of arbitrary geometry, however, is still a challenge [15].

The scaled boundary finite element method (SBFEM), a semi-analytical technique based on finite element technology, has been

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developed to model waves in unbounded domains [16]. This method combines some important features of the boundary element method and the finite element method. For example, only the boundary is discretised, which reduces the spatial dimension by one, while no fundamental solution is required. The SBFE equations are formulated from the governing equations by using the method of weighted residuals or the principle of virtual work. Anisotropy of the material only affects the constitutive matrix and is implemented straightforwardly. The scaled boundary finite element method has been successfully applied to wave propagation problems in unbounded domains in both frequency domain and time domain [17,18].

The scaled boundary finite element equation can be expressed in dynamic stiffness. This yields a system of non-linear first-order ordinary differential equations with respect to the frequency, which can be solved numerically. Applying the inverse Fourier transform to the scaled boundary finite element equation in dynamic stiffness leads to the scaled boundary finite element equation in unit-impulse response matrix in the time domain. This equation has to be solved using time-discretisation [19]. Originally, a time-discretisation scheme was proposed for the acceleration unit-impulse response matrix in the time domain assuming a piecewise constant variation within each time step. In order to obtain a stable solution, this method requires the whole time history to be discretised using a time step small enough such that the fastest wave travels less than the distance between two adjacent nodes in a time step. Linearisation of the acceleration unit-impulse response matrix has also been exploited [20–22]. Recently, a new time-discretisation scheme [23] based on the piecewise linear variation of the acceleration unit-impulse response matrix has been proposed. This new scheme improves the stability of the numerical approach by introducing an extrapolation parameter, so that a larger time step size can be used.

While the improved time-discretisation scheme [23] for the acceleration unit-impulse response has been shown to be very efficient, a similar algorithm for the displacement unit-impulse response matrix has not been proposed yet. Since the displacement unit-impulse response is approaching zero, a truncation time can be introduced and only the displacement unit-impulse response matrices before the truncation need to be calculated and processed in a convolution integral. This is equivalent to the linearisation techniques used in the context of acceleration unit-impulse response in Refs. [20–22]. The objective of this paper is to develop efficient algorithms for the calculation of the displacement unit-impulse response matrix and for the evaluation of the force–displacement relationship in the time domain. A piecewise linear variation is assumed and an extrapolation parameter is introduced to improve the stability of the scheme, so that larger time steps can be used in the time-discretisation. If smaller time steps are required in a time-domain analysis, intermediate values of the unit-impulse response matrix can be obtained by linear interpolation. Moreover, by truncating the displacement unit impulse response in the time-domain analysis, the computational effort is further reduced at a negligible loss of accuracy.

The further outline of the paper is as follows. The SBFEM equation in the frequency domain and the original time-discretisation scheme are summarised in Section 2. A new time-discretisation method for the displacement unit-impulse response matrix is given in Section 3. The coupling of far field and near field in a time-domain analysis is addressed in Section 4. Numerical examples are presented in Section 5. Conclusions are stated in Section 6.

2. Summary of the scaled boundary finite element method

The scaled boundary finite element method is described in Refs. [19,24,16]. For the sake of completeness, only a brief summary of the equations necessary for the development of the time-domain analysis in anisotropic media is given in this section.

In the scaled boundary finite element method, a scaling centre O is chosen in a zone from which the total boundary other than the straight surfaces passing through the scaling centre must be visible. The boundary is discretised using line elements for 2D problems and surface elements for 3D problems. The nodal unknown functions $\{u(\xi)\}$ are introduced along the radial lines passing through the scaling centre O and a node on the boundary, where ξ is the coordinate in the radial direction. In the frequency domain, using the method of weighted residuals in the circumferential directions (η, ζ), the SBFE equation in unknown function $\{u(\xi)\}$ results,

$$[E^0]\xi^2\{u(\xi)\}_{,\xi\xi} + ((s-1)[E^0] - [E^1] + [E^1]^T)\xi\{u(\xi)\}_{,\xi} + ((s-2)[E^1]^T - [E^2])\{u(\xi)\} + \omega^2[M^0]\xi^2\{u(\xi)\} = 0, \tag{1}$$

where s (=2 or 3) denotes the spatial dimension of the domain. In Eq. (1), zero body forces and prescribed surface tractions have been assumed. $[E^0]$, $[E^1]$, $[E^2]$ and $[M^0]$ are coefficient matrices obtained by assembling the element coefficient matrices as in the finite element method. They only depend on the geometry of the boundary and on the elasticity matrix, which can vary in the circumferential directions. Anisotropy can be modelled straightforwardly by using an appropriate constitutive matrix. The SBFE equation in displacement (1) can be transformed into an equivalent equation in dynamic stiffness:

$$([S^\infty(\omega)] + [E^1])[E^0]^{-1}([S^\infty(\omega)] + [E^1]^T) - (s-2)[S^\infty(\omega)] - \omega[S^\infty(\omega)]_{,\omega} - [E^2] + \omega^2[M^0] = 0. \tag{2}$$

Eq. (2) is a system of non-linear differential equations in the independent variable ω, where ω represents the angular frequency. The displacement dynamic stiffness matrix $[S^\infty(\omega)]$ can be changed into the acceleration dynamic stiffness matrix $[M^\infty(\omega)]$ by using $[M^\infty(\omega)](i\omega)^2 = [S^\infty(\omega)]$. In the time domain, inverse Fourier transform is applied to the SBFE equation in acceleration dynamic stiffness matrix, which yields a transformed SBFE equation in acceleration unit-impulse response matrix in the time domain:

$$\int_0^t [m^\infty(t-\tau)][m^\infty(\tau)]\,d\tau + t\int_0^t [m^\infty(\tau)]\,d\tau + [e^1]\int_0^t\int_0^\tau [m^\infty(\tau')]\,d\tau'\,d\tau + \int_0^t\int_0^\tau [m^\infty(\tau')]\,d\tau'\,d\tau[e^1]^T - \frac{t^3}{6}[e^2]H(t) - t[m^0]H(t) = 0, \tag{3}$$

where $[m^\infty(t)]$ is the transformed acceleration unit-impulse response matrix, after performing a Cholesky decomposition of $[E^0]$. A detailed derivation of Eq. (3) is given in Ref. [19].

The original time-discretisation scheme for Eq. (3) is based on the assumption of a piecewise constant variation of the acceleration unit-impulse response matrix with time. The constant value $[m^\infty]_n$ applies at time $t = (n-0.5)\Delta t$. Here, $n \geq 1$ and Δt is the time step size. An algebraic Riccati equation is obtained for the first time step, whereas the above approach leads to a Lyapunov equation for all the following time steps [18,19].

Numerical examples show that, when using the above constant discretisation scheme, in order to obtain a stable solution, the time step size Δt has to be chosen very small [23]. This leads to a large total number of time steps. Since Eq. (3) contains a convolution integral with respect to time, the large number of time steps will result in a large computational effort. In order to reduce the computational time, a new algorithm has been proposed in Ref. [23], which is based on assuming a piecewise linear variation of $[m^\infty]$ over each time step, and on using an extrapolation parameter θ to improve stability. That is, Eq. (3) is evaluated at time $t'_n = t_{n-1} + \theta\Delta t$ for each time station n. In the following section,