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A viscous-spring transmitting boundary for cylindrical wave propagation in saturated poroelastic media

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ABSTRACT

Based on the *u*–*U* formulation of Biot equation and the assumption of zero permeability coefficient, a viscous-spring transmitting boundary which is frequency independent is derived to simulate the cylindrical elastic wave propagation in unbounded saturated porous media. By this viscous-spring boundary the effective stress and pore fluid pressure on the truncated boundary of the numerical model are replaced by a set of spring, dashpot and mass elements, and its simplified form is also given. A *u*–*U* formulation FEA program is compiled and the proposed transmitting boundaries are incorporated therein. Numerical examples show that the proposed viscous-spring boundary and its simplified form can provide accurate results for cylindrical elastic wave propagation problems with low or intermediate values of permeability or frequency content. For general two dimensional wave propagation problems, spuriously reflected waves can be greatly suppressed and acceptable accuracy can still be achieved by placing the simplified boundary at relatively large distance from the wave source.

1. Introduction

The analysis of the dynamic behavior of saturated porous media is of great importance in geotechnical engineering and has attracted more and more attention in many fields. Particularly in earthquake engineering, the phenomena of liquefaction in soil can only be explained by considering the interaction of the soil skeleton and the pore fluid. The theory of wave propagation in saturated porous media was originally established by Biot [1] and has been developed by many researchers [2–5]. Numerical prediction such as finite element method based on concepts introduced by Biot has been widely used in solving dynamic problems due to its versatility and reliability. However, when the spatial domain of the problem is unbounded, a proper artificial boundary designed to simulate the wave propagation towards infinity without reflecting back is to be imposed on the truncated boundary of the finite element model. Such artificial boundary condition is also named in many literatures as absorbing, silent, non-reflecting, transmitting, radiating or transparent boundary conditions. The term ‘transmitting boundary’ is used here.

For one-phase media, a large number of transmitting boundaries have been proposed, a review on which can be found in [6] and the references therein. For saturated porous media, however, the interaction between the solid skeleton and the compressible pore fluid makes it much more complicated to establish the transmitting boundary for dynamic analysis.

Biot theory reveals that there are three kinds of body waves in saturated porous media. Inertial and mechanical coupling of the two phases makes $P1$ and $P2$ wave speeds not equal to that of single phase [7], whereas viscous coupling makes wave propagation dispersive. In condition of low permeability and low frequency, high viscous coupling makes the pore fluid absolutely restricted by solid skeleton and their relative motion disappears, thus, the saturated porous media behaves as a one-phase media. While in condition of high permeability and high frequency, viscous coupling no longer exists. Only in the above two extreme conditions are the body wave speeds non-dispersive. The transition from low to high viscous coupling behaviors occurs over quite a narrow range of permeability or frequency values [8].

Encouraged by the great advances achieved in one-phase media, several studies have been carried out on the transmitting boundaries for saturated porous media. Modaressi and Benzenati [9,10] first extended the paraxial approximation approach to the case of two-phase media assuming a plane wave pattern. By using Fourier transforms with respect to wave front direction and two tangential directions, the *u*–*p* equations are solved and three body wave speeds have been decomposed. For each wave the paraxial approximation method used in one-phase media can be directly adopted. In the same way, Akiyoshi et al. presented paraxial approximation with *u*–*w* formulations for linear isotropic porous media [11] and then for general conditions of transversely isotropic and anisotropic porous media [12]. The above transmitting

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boundaries are almost equivalent to viscous boundaries in the fundamental mode. To obtain the intensities of the dampers the second dilatational wave was neglected. By using Fourier transforms to solve the u–w equations, Degrande and De Roeck [13,14] have decomposed the solution of the wave field into incident and reflected waves and zeroed the amplitude of reflected wave in order to relate the effective stresses and pore pressures to the displacements of the transmitting boundary. Consequently the transmitting boundary is local in space and non-local in time, and the finite element system is solved in the frequency domain. Only in the low-frequency limit, it reduces to a frequency independent viscous boundary. Gajo et al. [15] established a first-order differential equation system which allow the propagation of elastic waves traveling only on the outgoing direction, then they obtained higher-order multidirectional boundaries by using the same generalizations proposed by Higdon [16,17]. The first-order boundary which can be viewed as an extension of the viscous method to two-phase media is derived for both cases of vanishingly small and infinitely large viscous coupling. The first condition corresponds to a very high permeability and high-frequency content, of which the results can be called the 'drained' boundaries. Whereas, the second condition corresponds to a low permeability and low-frequency content, of which the results can be called the 'undrained' boundaries. The advantage of this boundary lies in its independency of frequency and its convenience for implementation. For practical applications, suitable kinds of boundary condition can be selected according to the frequency content and the permeability value of the problem. Assuming an infinite permeability, Zerfa and Loret [18] developed a viscous boundary for transient analysis in the time domain. The method, derived from the constitutive equations proposed by Bowen's, consists of applying viscous tractions on both solid and fluid phases along the truncated boundary. The effect of the second dilatational wave is no longer neglected, and numerical results show that it works correctly also for problems with relatively low permeability.

The methods above are all based on one-dimensional plane wave hypothesis. As is noticed in one-phase media, the viscous boundary may have stability problem when dealing with low frequency dynamic loads. Cylindrical or spherical wave radiation should be considered in order to simulate the elastic recovery capability of the exterior saturated porous media. Based on the u–p cylindrical wave equations of saturated porous media with the assumption of zero permeability coefficient, Liu and Song [19] proposed a viscous-spring transmitting boundary, the derivation method of which is the same as Deeks and Randolph's method [20]. The efficiency and applicability of the proposed transmitting boundary were discussed. It was demonstrated that the proposed boundary can provide results with acceptable accuracy for earthquake engineering problems in saturated soils. Wang and Zhao [21] also developed two-dimensional and three-dimensional viscous-spring transmitting boundaries based on cylindrical and spherical u–U equations assuming an infinite permeability. The effect of the second dilatational wave has been neglected in both studies. It is worth noting that Li and Song [6] introduced the achievement of high-order accurate transmitting boundary to the application in saturated porous media. By using cylindrical wave equations and the assumption of zero permeability coefficient, the dynamic stiffness coefficients for cylindrical P and SV waves are constructed, then a temporal localization method is used to determine the spring, dashpot and mass parameters of the high-order local time-domain transmitting boundary. The approach is implemented into the DIANA SWANDYNE II program [22], and several different numerical examples demonstrated its good wave-absorbing capabilities.

The above transmitting boundaries for saturated porous media are all constructed based on the Biot theory [1], using different forms of governing equations, such as u–p, u–w and u–U formulations. The u–w and u–U formulations are equivalent to the original equations of motion proposed by Biot, while the u–p formulation is a simplification of the original equations, where the terms containing the fluid acceleration is neglected. As a consequence it is valid only for low and medium frequency problems. It is noticed that inaccuracy of the u–p formulation can be quite pronounced for high-frequency, short-duration problems [22]. Therefore development of transmitting boundary based on u–U equations is necessary.

In this paper, a time-domain viscous-spring transmitting boundary is derived from the cylindrical u–U equations, and its applications to both axi-symmetric and general plane-strain problems are demonstrated. The unbounded saturated porous media is assumed to be linear elastic and isotropic and the assumption of zero permeability coefficient is also applied. The effective stresses on the truncated boundary results from the combination of a line of spring, dashpot, mass elements linked to the solid phase on both of the radial and the circumferential directions, while the pore fluid pressure results from the combination of a line of dashpot, mass elements linked to the fluid phase on the radial directions. A u–U formulation FEA program is compiled and the proposed viscous-spring transmitting boundary as well as its simplified form are implemented therein. The accuracies of the proposed transmitting boundaries are demonstrated by numerical results for cylindrical radiation problems in saturated porous media. The potential applications of the simplified boundary to general two-dimensional infinite wave radiation problems are also discussed.

2. Governing equations

After the establishment of the theory of propagation of elastic waves in saturated porous media by Biot [1], many derivations and modifications have been made subsequently. The generally used u–U forms of equations are proposed by Zienkiewicz and Shiomi [4]. For small strain linear elastic isotropic porous media the governing equations are as follows:

$$\sigma_{ij,j}+\rho b_i=\rho_1\ddot{u}_i+\rho_2\ddot{U}_i \tag{1}$$

$$-p_{,i}+\rho_f b_i=\rho_f\ddot{U}_i+nk_f^{-1}(\dot{U}_i-\dot{u}_i) \tag{2}$$

$$\sigma_{ij}=\sigma'_{ij}-\alpha\delta_{ij}p=2\mu\varepsilon_{ij}+\lambda\varepsilon_{kk}\delta_{ij}-\alpha\delta_{ij}p \tag{3}$$

$$p=-Q((\alpha-n)\varepsilon_{ii}+nU_{i,i}) \tag{4}$$

where u_i and U_i are the absolute displacement of the solid and the pore fluid respectively; σ_{ij} and σ'_{ij} are the total stress and effective stress respectively; b_i is the body force; p is the pore fluid pressure; n is the porosity of the soil; ρ, ρ_s and ρ_f are densities of the assembly, the solid and fluid phases, respectively and $\rho_1=(1-n)\rho_s$, $\rho_2=n\rho_f$, $\rho=\rho_1+\rho_2$; k_f is the permeability which is related with Darcy permeability coefficient k by $k_f=k/\rho_f g$ in which g is the gravity acceleration; λ and μ are Lame constants for the solid skeleton; α and Q are two introduced parameters, which can be expressed as

$$\begin{aligned}\alpha&=1-K_b/K_s\\ 1/Q&=n/K_f+(\alpha-n)/K_s\end{aligned} \tag{5}$$

where K_s, K_f and K_b are the bulk modulus of the solid, fluid and the assembly respectively.