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Rectangular footing on soil with depth-degrading stiffness: Vertical and rocking impedances under conditional existence of surface waves

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ABSTRACT

The response of rectangular rigid footings resting on an elastic soil of shear modulus decreasing monotonically with depth is studied. Such profiles are typically encountered after ground improvement. The propagation characteristics of SV/P surface waves are investigated, showing the appearance of cut-off frequencies above which surface waves do not exist. The semi-analytical method of the subdivision of the footing/soil contact area is then used for solving the boundary value problem, whereby the influence functions for the sub-regions are determined by integration of the corresponding surface-to-surface Green's functions. Impedance functions are presented over a wide range of frequencies for typical values of the non-homogeneity parameters, the Poisson's ratio and the foundation geometry. The salient features that are associated with the non-homogeneity and the appearance of cut-off frequencies are elucidated.

1. Introduction

Elastic solids with shear modulus varying with depth are successfully used to model wave propagation and the associated phenomena in geomechanics. While stratified media present the general case, the continuous variation of stiffness with depth is characteristic to uniformly layered deposits due to the dependency of soil stiffness on effective confining pressure. The continuous variation of soil stiffness with depth is often referred to as non-homogeneity. The vast majority of work hitherto published deals with a monotonic increase of shear modulus or shear wave velocity with depth. In analytical solutions the criterion for the selection of a particular profile was the solvability of the associated mathematical problem. Often, a specific value of the Poisson's ratio is assigned to the half-space to simplify matters. Among the various relevant publications, one may mention those by Awojobi [1], Selvadurai et al. [2], Waas et al. [3], Guzina and Pak [4], Vrettos [5], Muravskii [6], Baziar and Song [7]. A summary is given by Mylonakis et al. [8]. At the same time, the case of a monotonic decrease of soil stiffness with depth has attracted less attention, despite the fact that there are situations where either the natural ground exhibits a stiff crust, or the surficial layers have been strengthened by ground improvement techniques to provide adequate foundation bearing [9].

The case of depth-degrading soil stiffness shows some particularities that are not obvious at first sight. While it can be easily shown that SH surface wave propagation is not possible in this type of half-space, as known from the derivation for Love waves in a layer underlain by a homogeneous half-space, systematic investigation of the conditions for the existence of SV/P surface waves in a half-space with depth-decreasing modulus is limited. In a paper by the author [10], it has been shown that for a sufficiently strong vertical non-homogeneity, SV/P surface waves (the counterpart of the Rayleigh wave for nonhomogeneous profiles) do not exist above certain frequencies. Consequently, the associated surface-to-surface Green's functions (half-space displacement due to unit surface load) show at large distance from the source the attenuation typical for compressional and shear body waves [10].

Limited information for this class of soil pertains also to the impedance functions for rigid footings, an exception being the work by Gazetas [11] that presents impedance functions for rigid strip footings on a two-layer system consisting of a surface layer with parabolically decreasing stiffness underlain by a half-space. The static torsional response has been investigated by Rajapakse and Selvadurai [12] by assuming a layer with linearly decreasing stiffness underlain by a half-space with linearly increasing stiffness with depth.

To fill this gap, impedance functions for rectangular footings for vertical and rocking motion are derived in the following. The method of the subdivision of the contact area is adopted for the solution by assuming relaxed boundary conditions for the soil-footing interface.

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The Green's functions developed by the author for a reverse-type of vertical non-homogeneity with finite modulus values at the surface and at large depths are used in the respective numerical method.

2. Problem statement

Consider a rigid massless rectangular foundation with side lengths $2b$ and $2a$ with $b \geq a$ resting on the surface of a linear–elastic, isotropic half-space of constant mass density ρ and the Poisson's ratio v, with $0 \leq v < 0.5$, and shear modulus G varying with depth z such that

$$G(z) = G_0 + (G_\infty - G_0)(1 - e^{-\alpha z}) \tag{1}$$

where G_0 and G_∞ are the shear moduli at the surface and at infinite depth, respectively, and α is a constant with dimension of inverse length which is referred to as non-homogeneity gradient. $G_0 \leq G_\infty$ corresponds to the regular case encountered in soils, while a depth-degrading modulus is described by simply setting $G_0 \geq G_\infty$.

The foundation is loaded at its center by a harmonic vertical load $Pe^{i\omega t}$ and harmonic moments $M_y e^{i\omega t}$ and $M_x e^{i\omega t}$ about the long and short foundation axis, respectively, as depicted in Fig. 1, whereby ω is the circular frequency, i is the imaginary unit, and t denotes time. The contact area is assumed to be frictionless, i.e. only normal stresses and vertical displacements are considered (relaxed boundary conditions).

The method adopted for solving this mixed boundary value problem is identical to that outlined in Refs. [5,13]. Before proceeding with that solution, the wave propagation characteristics in this type of soil are first considered.

3. SV/P surface wave propagation

Surface waves under free-field conditions require traction-free surface and vanishing displacements at large depths. These conditions lead to the characteristic equation of the respective eigenvalue problem. For a homogeneous half-space, and depending on the value of the Poisson's ratio, the characteristic equation has either three real roots or one real root and two complex conjugate roots. The smallest real root defines the Rayleigh surface wave that propagates along the surface and decays into the medium. The other roots are usually classified as physically not meaningful [14]. Hence, only one vibration mode, the fundamental mode, is possible.

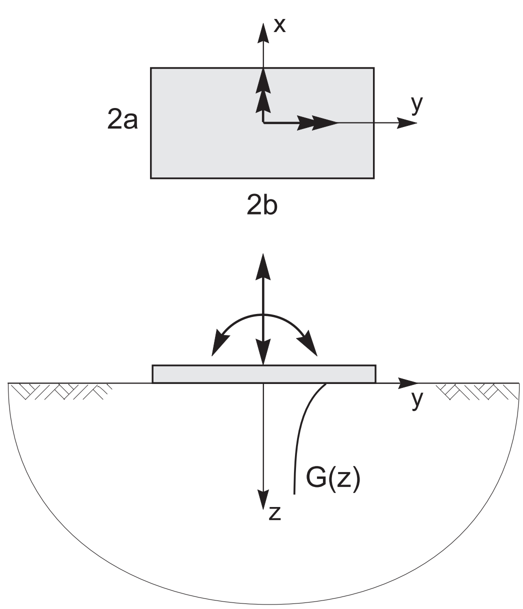

Fig. 1. The problem under consideration.

For a half-space with shear modulus varying with depth, the solution of the characteristic equation is expressed in terms of eigenvalue pairs of wavelength versus frequency (k, ω) or, equivalently, propagation velocity vs. frequency $(v_{SV/P}, \omega)$. This relationship defines the dispersion law of the medium.

For the regular case of depth-*increasing* modulus, several vibration modes are possible: the fundamental mode that appears over the entire frequency range, and higher modes that arise above distinct cut-off frequencies [15]. The dispersion is called normal with the propagation velocity decreasing with frequency.

Things are entirely different when the modulus *decreases* with depth: When solving the eigenvalue problem for a half-space with $G_0 > G_\infty$, it is observed that as long as the non-homogeneity is weak, a real-valued solution of the characteristic equation is found over the entire frequency range. The dispersion is characterized as anomalous with higher frequencies yielding higher surface wave propagation velocities. However, as the non-homogeneity becomes stronger, and depending on the value of the Poisson's ratio, cut-off frequencies appear above which no real valued wave number is found as solution to the characteristic equation, i.e. no solution fulfilling the radiation condition at infinity is possible [10]. The derived solution is valid for

$$\frac{G_0}{G_\infty} < 2 \tag{2}$$

whereby for convenience the moduli ratio is replaced by the degree of non-homogeneity Ξ_0^* :

$$\Xi_0^* = 1 - \frac{G_\infty}{G_0} \tag{3}$$

Numerical values for the cut-off frequencies for typical values of Ξ_0^* and of the Poisson's ratio v are given in Table 1. They are slightly different from those given in Ref. [10] due to the increased accuracy near the cut-off limit. The frequency is given in dimensionless form in terms of the parameter $\overline{\theta}$, which links the non-homogeneity gradient α to the shear wave number corresponding to the shear modulus at the surface G_0

$$\overline{\theta} = \frac{\omega}{\alpha v_{S0}} \tag{4}$$

where

$$v_{S0} = \sqrt{\frac{G_0}{\rho}} \tag{5}$$

Table 1
Cut-off frequencies $\overline{\theta}_{\text{cut-off}}$ for the existence of SV/P surface waves for various combinations $0.2 \leq \Xi_0^* \leq 0.49$ and $0 \leq v \leq 0.49$. A hyphen indicates that a cut-off frequency was not found.

v	$\overline{\theta}_{\text{cut-off}}$ Ξ_0^*					
	0.2	0.25	0.3	0.4	0.44	0.49
0	–	22.565	4.893	1.813	1.402	1.055
0.05	–	10.233	3.893	1.654	1.301	0.992
0.1	–	6.601	3.239	1.518	1.211	0.934
0.15	22.277	4.949	2.777	1.401	1.130	0.880
0.2	10.355	3.947	2.431	1.296	1.056	0.829
0.25	6.765	3.287	2.159	1.203	0.987	0.781
0.3	5.030	2.817	1.938	1.117	0.924	0.735
0.35	4.008	2.461	1.752	1.038	0.864	0.691
0.4	3.319	2.178	1.591	0.965	0.807	0.649
0.45	2.844	1.944	1.449	0.895	0.752	0.607
0.49	2.538	1.781	1.345	0.842	0.710	0.575