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Formulation of a sand plasticity plane-strain model for earthquake engineering applications

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ABSTRACT

The formulation of a sand plasticity model for geotechnical earthquake engineering applications is presented. The model follows the basic framework of the stress-ratio controlled, critical state compatible, bounding surface plasticity model for sand presented by Dafalias and Manzari, Journal of Engineering Mechanics, ASCE, 2004;130(6): 622–634. . Modifications to the model were developed and implemented to improve its ability to approximate the stress–strain responses important to geotechnical earthquake engineering applications. These constitutive modifications included: revising the fabric formation function to depend on plastic shear rather than plastic volumetric strains; adding fabric history and cumulative fabric formation terms; modifying the plastic modulus relationship and making it dependent on fabric; modifying the dilatancy relationships to provide more distinct control of volumetric contraction versus expansion behavior; providing a constraint on the dilatancy during volumetric expansion so that it is consistent with Bolton′s Géotechnique 1986; 36(1): 65–78. dilatancy relationship; modifying the elastic modulus relationship to include dependence on stress ratio and fabric history; modifying the logic for tracking previous initial back-stress ratios (i.e., loading history effect); recasting the critical state framework to be in terms of a relative state parameter index; and simplifying the formulation by restraining it to plane strain without Lode angle dependency for the bounding and dilation surfaces. Model responses to various loading conditions, including drained and undrained monotonic and cyclic loading, are used to illustrate the efficacy of the model modifications. Calibration and implementation of the model for practice are described in a companion paper.

1. Introduction

Constitutive models for sand in earthquake engineering applications range from relatively simplified, uncoupled cycle-counting models to more complex plasticity models (e.g., Wang et al. [3], Cubrinovski and Ishihara [4], Dawson et al. [5], Papadimitriou et al. [6], Yang et al. [7], Byrne et al. [8], Dafalias and Manzari [1]). The utility of a constitutive model in practice depends on its ability to approximate the range of soil behaviors that are important to the application at hand and the level of effort required to calibrate the model with the types of data available.

Constitutive models for many geotechnical earthquake engineering applications have to approximate a broad mix of conditions in the field. For example, a single geotechnical structure like an earth dam (Fig. 1) can have strata or zones of sand ranging from very loose to dense, under a wide range of confining stresses, initial static shear stresses (e.g., at different points beneath the slope), drainage conditions (e.g., above and below the water table), and loading conditions (e.g., various levels of shaking). The engineering effort is greatly reduced if the constitutive model can reasonably approximate the predicted stress–strain behaviors under all these different conditions. If the model cannot approximate the trends across all these conditions, then extra engineering effort is required in deciding what behaviors should be prioritized in the calibration process, and sometimes by the need to repeat the calibrations for the effects of different initial stress conditions within the same geotechnical structure.

The data available for the characterization of sand in design practice most commonly include basic classification index tests (e.g., grain size distributions), penetration resistances (e.g., SPT or CPT), and shear wave velocity (V_s) measurements. More detailed laboratory tests, such as triaxial or direct simple shear (DSS) tests, are almost never available due to the problems with overcoming sampling disturbance effects and the challenge of identifying representative samples from highly heterogeneous deposits.

Sand models are therefore almost solely calibrated against empirical design correlations in practice. It is unlikely that any one sand model can be developed or calibrated to simultaneously fit a full set of applicable design correlations for monotonic and cyclic, drained and undrained behaviors of sand, in part because

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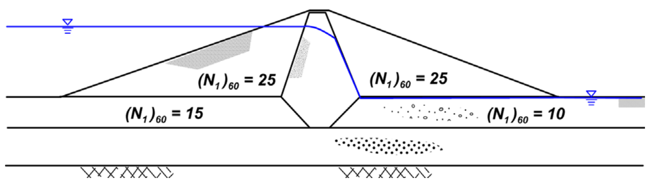

Fig. 1. Cross-section of an earth dam illustrating the wide range of density, saturation, and stress conditions that may need to be modeled.

the various design correlations are not necessarily physically consistent with each other; e.g., they may include a mix of laboratory test-based and case history-based relationships, or they have been empirically derived from laboratory data sets for different sands. Nonetheless, it is desirable that a model, after calibration to the design relationship that is of primary importance to a specific project, be able to produce behaviors that are reasonably consistent with the general magnitudes and trends in other applicable design correlations or typical experimental data.

Stress–strain behaviors of sand that are commonly the focus in design include:

- The small-strain shear modulus which can be obtained through in-situ shear wave velocity measurements.
- The shear modulus reduction and equivalent damping ratio relationships which are commonly estimated using empirical correlations.
- The cyclic resistance ratio (CRR) against triggering of liquefaction which is commonly estimated based on SPT- and CPT-based liquefaction triggering correlations.
- The response under irregular cyclic loading histories of varying duration, which relates to the magnitude scaling factors (MSF) that are used with liquefaction correlations in practice.
- The dependence of CRR on effective confining stresses and sustained static shear stresses, which are represented by the K_σ and K_α correction factors in practice.
- The accumulation of shear strains after triggering of liquefaction, which can only be evaluated relative to typical data in the literature.
- Strength loss as a consequence of liquefaction, which may involve explicitly modeling phenomena such as void redistribution or empirically accounting for it through case history-based residual strength correlations.
- Drained monotonic shear strengths (or peak friction angles) and undrained monotonic shear strengths, which may be estimated using correlations to SPT and CPT penetration resistances.
- Volumetric strains during drained cyclic loading or due to reconsolidation following triggering of liquefaction, both of which may be estimated using empirical correlations derived from laboratory test results for similar soils in the literature.

The formulation of a plasticity model for sand (referred to as PM4Sand) for geotechnical earthquake engineering applications is presented. The PM4Sand model follows the basic framework of the stress-ratio controlled, critical state compatible, bounding surface plasticity model for sand initially presented by Manzari and Dafalias [9] and later extended by Dafalias and Manzari [1]. The Dafalias–Manzari model, with its state-parameter dependent dilatancy and fabric-dilatancy tensor features, provides a flexible framework for modeling behavior of sands over a wide range of densities and confining stresses. Modifications to the Dafalias–Manzari model were developed and implemented by Boulanger [10] and further herein to improve its ability to approximate a set of engineering design relationships that are used to estimate the stress–strain behaviors that are important to predicting liquefaction-induced ground deformations during earthquakes. In effect, the approach taken was to calibrate the constitutive model at the equation level, such that the functional forms for the various constitutive relationships were chosen for their ability to approximate the important trends embodied in the extensive laboratory-based and case history-based empirical correlations that are used in practice. This paper presents the model formulation, the motivations for the selected functional forms, and examples of the capabilities provided by the various modifications and additions to the model. The companion paper (Ziotopoulou and Boulanger [11]) describes the calibration of the model against a set of engineering design correlations and the numerical implementation of the sand model as a dynamic link library (DLL) for use with a two-dimensional explicit finite difference program.

2. Model background

The sand plasticity model presented herein follows the basic framework of the stress-ratio controlled, critical state compatible, bounding-surface plasticity model for sand presented by Dafalias and Manzari [1]. The Dafalias and Manzari [1] model extended the previous work by Manzari and Dafalias [9] by adding a fabric-dilatancy related tensor quantity to account for the effect of fabric changes during loading. The fabric-dilatancy related tensor was used to macroscopically model the effect that microscopically-observed changes in sand fabric during plastic dilation have on the contractive response upon reversal of loading direction. Dafalias and Manzari [1] provide a detailed description of the motivation for the model framework, beginning with a triaxial formulation that simplifies its presentation and then followed by a multi-axial formulation. The model proposed herein is presented in its multi-axial formulation, along with the original framework of the Dafalias–Manzari model for comparison.

The constitutive equations for the model presented herein are summarized in Table 1 along with the equations for the Dafalias–Manzari [1] model. Detailed descriptions of all model terms are provided in Boulanger and Ziotopoulou [13], along with simulated single element responses for a broad range of conditions.

3. Basic stress and strain terms

The model is based on effective stresses, with the conventional prime symbol dropped from the stress terms for convenience. Stresses are represented by the tensor $\boldsymbol{\sigma}$, the mean effective stress p, the deviatoric stress tensor $\boldsymbol{s}=\boldsymbol{\sigma}-p\boldsymbol{I}$ (with $\boldsymbol{I}$=identity matrix), and the deviatoric stress ratio tensor $\boldsymbol{r}=\boldsymbol{s}/p$. Computational speed was improved by limiting the implementation to plane strain conditions, assuming that soil properties (e.g., moduli) are functions of the mean of the in-plane stresses alone, and then condensing out the computation of the out-of-plane stress.

The model strains are represented by a tensor $\boldsymbol{\varepsilon}$, which can be decomposed into the volumetric strain ε_v and the deviatoric strain tensor $\mathbf{e}$. The volumetric strain is the trace of $\boldsymbol{\varepsilon}$ and the deviatoric strain tensor is $\boldsymbol{e}=\boldsymbol{\varepsilon}-(\varepsilon_v/3)\boldsymbol{I}$. The deviatoric and volumetric strain increments are further decomposed into elastic and plastic components,

$$d\boldsymbol{e}=d\mathbf{e}^e+d\mathbf{e}^p \tag{1}$$

$$d\varepsilon_v=d\varepsilon_v^e+d\varepsilon_v^p \tag{2}$$

where $\boldsymbol{e}^e$=elastic deviatoric strain tensor, $\boldsymbol{e}^p$=plastic deviatoric strain tensor, $\varepsilon_v{}^e$=elastic volumetric strain, and $\varepsilon_v{}^p$=plastic volumetric strain. The elastic deviatoric strain and elastic volumetric strain are computed as,

$$d\mathbf{e}^e=\frac{d\mathbf{s}}{2G} \tag{3}$$