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New high-precision empirical methods for predicting the seepage discharges and free surface locations of earth dams validated by numerical analyses using the IFDM

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Abstract

The interpolation finite difference method (IFDM) can be used in the numerical analysis of steady-state seepage problems over arbitrary domains. In such analyses, by providing the coordinates of change points considering the polygonal domain and boundary conditions, a grid is automatically generated and a numerical solution is promptly obtained. Homogeneous, isotropic dams without tail drains have been investigated in a previous study. However, to consider dams with tail drains, we must alter the previous system slightly. This paper describes the altered system. In earlier times, when computation power was limited and before modern numerical calculation methods had been established, it was highly important for dam engineers to estimate the seepage discharge and the location of the free surface. Several empirical methods were proposed, and these have retained their functionality. For instance, the basic parabola method is an exemplary work that builds on classic empirical methods. In this work, the results obtained using A. Casagrande's method are compared with those obtained by IFDM, and it is confirmed that there are considerable deviations between them. The so-called "equivalent KZ flow method" is proposed and shown to significantly improve the calculation accuracy for the discharge and free surface location.

Keywords: Seepage analysis; Interpolation FDM; Arbitrary domain; Flow net; Basic parabola; Equivalent KZ flow method

1. Introduction

The finite difference method (FDM) is the oldest numerical approach for solving partial differential equations (PDEs). It is very simple and effective for structured grids, and higher-order schemes, especially, can be easily obtained on regular grids. However, FDM has the disadvantage that certain conservation laws are not enforced unless special steps are taken. Furthermore, it has the significant disadvantage of being restricted to simple geometries, and thus cannot be applied to complex flows (Ferziger and Perić, 2002). However, an exploratory approach to overcome such disadvantages has been reported in the literature (Shortley and Weller, 1938). The author has systematically studied this problem and reported the basic concept of the interpolation finite difference method (IFDM) (Fukuchi, 2011, 2013, 2014). Various calculation methods based on FDM over regular domains have been proposed. All of these accumulated for a long time, would be progressively applied to the calculation over irregular domains by IFDM. It is expected that the IFDM theory will be established as a comprehensive numerical system wherein the validity of the theory must be confirmed individually for each specific problem.

In a previous paper (Fukuchi, 2016), calculation methods related to confined and unconfined seepage problems were formulated. In recent years, the so called "Cartesian

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Nomenclature

A_e	equivalent area, $A_e = A_k - A_{de}$
A_k	total area of the basic KZ flow
A_{de}	deficit area
BPP	boundary potential parabola
C_j	convergence judgment factor
FS	free surface
FSP	free surface parabola
KZ flow	Kozeny flow
KZb(C, O)	basic KZ flow (determined by points C and A)
KZc(A, O)	criteria KZ flow
KZ1(C, A_e)	area equivalent KZ flow
KZ2(C, B_n)	discharge-point equivalent KZ flow
KZ3(C, q_t)	discharge equivalent KZ flow
KZn(q_n, B_n)	numeric equivalent KZ flow
L_h^*	dimensionless half-decrease distance
$l_{wi,j}$	wall-distance factor
m_v	C-point determination factor.
point A	entrance point
point B	discharge point
point B_n	discharge point defined by numerical calculation
point C	starting point of the basic parabola
point C_a	starting point of the basic parabola after A. Casagrande
point C_n	starting point defined by numeric equivalent KZ flow
point O	origin of KZ coordinate system
q_{Ca}	discharge calculated from A. Casagrande's method
q_{LC}	discharge calculated from L. Casagrande's method
q_n	numerically calculated discharge
q_{SV}	discharge calculated from Schaffernak–Van Iterson's method
q_t	theoretical discharge
α	discharge angle
α_b	acceleration factor of the TMSD scheme
θ	entrance angle
θ_c	criteria entrance angle
$\varphi_{i,j}$	time-interval-adjustment factor

grid methods" (CGMs) have been used in the numerical analysis of continuum physical phenomena. CGMs have considerable merits in the generation of calculation grids and the formulation of numerical calculation methods (Aftosmis et al., 1998; Fukuchi, 2014). The proposed method to solve seepage problems is a CGM with sufficient generality to solve Laplace equation. This method, if required, can be naturally developed for higher-order accurate calculations. In the previous study (Fukuchi, 2016), a calculation method for dams without tail drains was formulated; in this paper, a general calculation method, regardless of the presence or absence of tail drains, is presented.

IFDM can be broadly classified into the boundary polynomial interpolation method (BPIM) and algebraic polynomial interpolation method (APIM) (Fukuchi, 2014). In this and the previous study (Fukuchi, 2016), the numerical calculations are conducted using IFDM-BPIM. The basis of these methods is that any calculation over irregular domains is simplified such that Dirichlet problems can be numerically solved over a regular domain by adopting a boundary polynomial interpolation. The same process is applied for seepage problems with the Dirichlet–Neumann mixed boundary conditions. In the calculation, the second-order accurate centered space difference is used, and the boundary interpolation equation is quadratic. This scheme is defined as the O2B2 scheme. The BPIM scheme is generally expressed as OmBn ($m = 2, 4, \ldots, n = 0, 1, 2, \ldots$), and the O2B2 scheme is the lowest accurate-order scheme under the condition that $m = n$. Its accuracy is sufficient and comparable to that of the calculation results obtained using other techniques such as the finite element method (FEM) and boundary element method (BEM) (Fukuchi, 2016).

With rapid numerical calculations of unconfined seepage problems, the systematic evaluation of empirical methods for determining the discharges and free surface locations of seepage problems becomes possible. In the first half of the last century, numerous (semi)-theoretical empirical methods were proposed. Among these, there are four well-known empirical methods (Casagrande, 1937): (i) Schaffernak–Van Iterson's method, (ii) L. Casagrande's method, (iii) Kozeny's method, and (iv) A. Casagrande's method. In these methods, the calculation results obtained using A. Casagrande's methods are compared with the results of IFDM-BPIM calculations. His method is adopted in various design manuals (EMED, 1993; USDIBR, 2014) and is still in use. In the appendices, the theories of methods (iii) and (iv), which are focused in the paper, are described in detail.

A. Casagrande's method is the most comprehensive empirical approach; however, some essential problems are apparent. The method is based on Kozeny's method, but the formulation contradicts the essential concept of Kozeny flow (Appendix A.1). In this paper, several kinds of Kozeny flows are introduced beyond the concept of the seepage phenomenon investigated by Kozeny (1931); therefore, from now on, let us express the term, Kozeny flow, as the KZ flow (Appendix A.1). Based on precise investigations of the KZ flow, an alternative "equivalent