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On an implicit stress-calculation algorithm for a multidimensional constitutive law using a skeleton curve

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Abstract

In this paper, we present an implicit stress-calculation algorithm for a multidimensional constitutive law using a one-dimensional skeleton curve and a hysteresis curve. To consider the hysteretic behavior of soils, one-dimension skeleton curves (e.g., the Hardin–Drnevich model, Ramberg–Osgood (RO) model, and general hyperbolic equation (GHE) model) are widely used. In a multidimensional analysis, there are several methods for extending a constitutive law using a skeleton curve into multiple dimensions. However, because these methods are suitable for an explicit integration stress-calculation scheme, a calculated stress does not satisfy the function of skeleton curve when a large incremental strain is imposed in one step. In this paper, to resolve this problem, a nonlinear stress–strain relation using a skeleton curve and an implicit stress-calculation algorithm is presented. In addition, we show an example of an application of the GHE/RO model to the proposed algorithm.

Keywords: Skeleton curve; Multidimension; Implicit stress-calculation algorithm; GHE model; RO model

1. Introduction

Accurate modeling of the nonlinear stress–strain relation of soils is required for a seismic-response analysis. To express a nonlinear stress–strain relation, the Ramberg–Osgood (RO) model (Ramberg and Osgood, 1943), Hardin–Drnevich (HD) model (Hardin and Drnevich, 1972), and general hyperbolic equation (GHE) model (Tatsuoka and Shibuya, 1991) are widely used in seismic-response analyses. These models can express the hysteretic behavior of soils as two one-dimensional curves, as shown in Fig. 1. One curve is related to monotonic-loading, known as the skeleton curve, and the other curve is related to unloading/reloading conditions, known as the hysteresis curve. To use a skeleton curve and a hysteresis curve in multiple dimensions, a constitutive law must be extended multidimensionally. There are several methods which can be used to extend a constitutive law using a skeleton curve. The most-simple multidimensional constitutive law uses a skeleton curve only for shear stress and shear strain. However, this constitutive law is obviously not objective, since it cannot express hysteretic behavior due to an axial strain.

Chitas et al. (2012), Yoshida (2006), and Yoshida and Tsujino (1993) used equivalent deviatoric stresses and strains to extend constitutive laws to multiple dimensions.

Chitas et al. (2012) presented an incremental stress–strain relation that uses a secant shear modulus Gs and a secant bulk modulus Ks. Yoshida (2006), Yoshida and Tsujino (1993) presented another incremental formulation that uses a tangent stiffness $d\eta/d\xi$ of the skeleton curve (Yoshida method). Fig. 2 shows the relation between a skeleton curve and a tangent stiffness in one-dimension. Note that ξ, η, Δ, and n are the nondimensional equivalent deviatoric strain, nondimensional equivalent deviatoric stress, incremental value, and number of steps, respectively.

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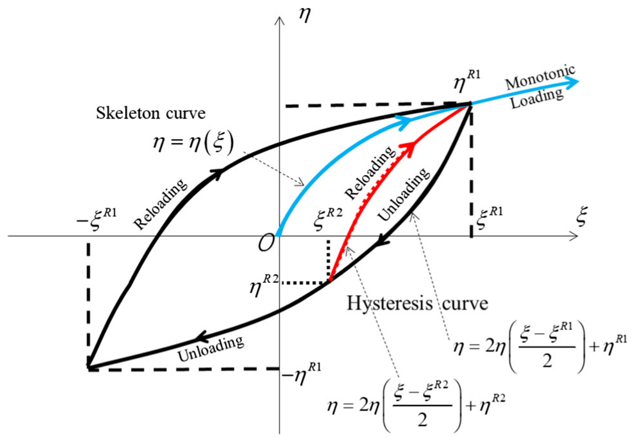

Fig. 1. Skeleton and hysteresis curves using the Masing rule in one-dimension.

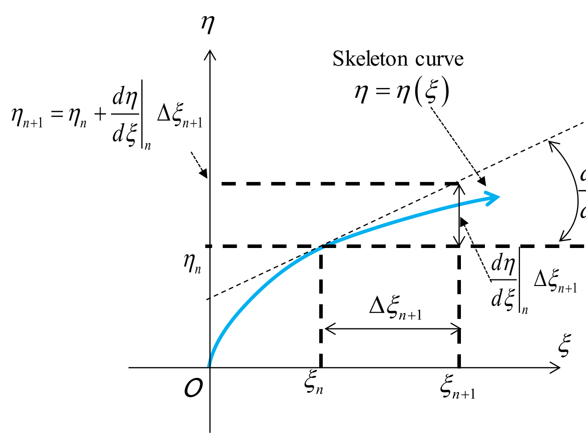

Fig. 2. Explicit stress–calculation.

From Fig. 2, a stress η_{n+1} is obtained as $\eta_{n+1} = \eta_n + (d\eta/d\xi|_n)\Delta\xi_{n+1}$. Obviously, in multiple dimensions, a tensor notation is used as $\boldsymbol{\eta}_{n+1} = \boldsymbol{\eta}_n + (d\boldsymbol{\eta}/d\boldsymbol{\xi}|_n)\Delta\boldsymbol{\xi}_{n+1}$. This stress-calculation method takes account of the cyclic behavior due to axial strain. In addition, Yoshida (2006), Yoshida and Tsujino (1993) clarified the unloading and loading conditions using $\boldsymbol{\eta}_{n+1}$ and "stack of reversals" as described later. Therefore, the Yoshida method is a very simple and useful algorithm for a seismic-response analysis.

However, since these above methods use incremental values to calculate stress $\boldsymbol{\eta}_{n+1}$, a step dependency appears in these methods when using a strongly nonlinear skeleton curve or when a large strain is imposed in one step.

Hueckel and Nova (1979) and Niemunis et al. (2011) presented a paraelastic stress–strain relation using a stress span, $\mathbf{S} - \mathbf{S}^{Ri}$, and a strain span, $\mathbf{e} - \mathbf{e}^{Ri}$, where Ri denotes a reversal point. Niemunis et al. (2011) used a "stack of reversals" to clarify unloading and loading conditions. This stack of reversals is a robust method for expressing loading conditions. However, Niemunis et al. (2011) pointed out that a large incremental strain may lead to inconsistencies in the constitutive description, i.e., a step dependency also appears in this method. They also pointed out that various skeleton curves exist, but their 3D generalizations were not presented.

We believe that an explicit expression of one-to-one correspondence between stress and strain without resort to incremental or rate variables and an algorithm for determining loading conditions are necessary to resolve this step dependency.

Therefore, in this paper, assuming that a constitutive law has a coaxial stress–strain relation, we find a nonlinear stress–strain relation using stress span $\mathbf{S} - \mathbf{S}^{Ri}$, strain span $\mathbf{e} - \mathbf{e}^{Ri}$, and skeleton curve $\eta = \eta(\xi)$. Subsequently, we present that an implicit stress-calculation algorithm including a determination algorithm of loading conditions using the obtained nonlinear stress–strain relation. The proposed algorithm constitutes an improved version of the Yoshida method.

In particular, an explicitly nonlinear stress–strain expression of the GHE model is obtained. The obtained nonlinear stress–strain relation does not use incremental values. Moreover, in the GHE model, a consistent tangent modulus for the Newton-Raphson method can be obtained.

In addition, we show that, even if a skeleton curve is defined by a nonlinear function that cannot be transformed into $\eta = \eta(\xi)$ as in the RO model, the calculated stress also has step independence.

Finally, we show examples of calculation using the proposed algorithm. First is the example of the GHE model to validate the proposed algorithm. Second, a modified RO model is used to show that the proposed algorithm can obtain a step-independent stress in a skeleton curve that cannot be transformed into $\eta = \eta(\xi)$. Then, a pulsating cyclic strain and an alternate cyclic strain are used to show the validity of the proposed algorithm, as in a seismic-response analysis.

2. Nonlinear stress–strain relation using a skeleton curve

Consider an incremental initial-value problem at the time interval $[t_n, t_{n+1}]$. Here, stress is expressed by a hydrostatic stress (i.e., mean pressure), $p(t_{n+1}) := -(\mathrm{tr}\boldsymbol{\sigma})/3$, and a deviatoric stress, $\mathbf{S}(t_{n+1})$, as below:

$$\boldsymbol{\sigma}(t_{n+1}) = -p(t_{n+1})\mathbf{1} + \mathbf{S}(t_{n+1}), \tag{1}$$

where $\mathbf{1}$ is the second-order identity tensor. Assuming that the volumetric component has no cyclic behavior and no history dependence, p becomes

$$p(t_{n+1}) = K\varepsilon_v(t_{n+1}), \tag{2}$$

where $\varepsilon_v := -\mathrm{tr}\boldsymbol{\varepsilon}$ is the volumetric strain and K is the bulk modulus, which is held constant to simplify this discussion.

On the other hand, since deviatoric stress is history-dependent, it cannot be simply obtained. Therefore, in this paper, two loading conditions are considered to calculate the stress. One loading condition is a monotonic-loading