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Velocity-based time-discontinuous Galerkin space-time finite element method for elastodynamics

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Abstract

A velocity-based single-field Space-Time Finite Element Method (v-ST/FEM) is devised within the framework of the time-discontinuous Galerkin method for an elastodynamics problem. The new method uses finite elements for both space and time domains, and reduces the size of the resulting linear system to be solved in each time step compared to the two-field formulation. In v-ST/FEM, the trial functions for the velocity field are continuous in space and discontinuous in time, while the test functions are continuous in both space and time. The displacement and stress fields are computed in a post-processing step using the time-integration process which explicitly includes the velocity field. Accordingly, the displacement-velocity compatibility condition and the continuity of the displacement field in time are strongly imposed in the displacement-velocity-based two-field formulation. In this way, a velocity-based single-field weak formulation is derived from the two-field formulation. The present method is found to be unconditionally stable and third-order accurate. Following a review of the space–time finite element literature, the general theoretical development and formulation aspects of the present methodology are demonstrated. Several numerical examples are given to show the computational performance of the proposed scheme.

Keywords: Space-Time FEM; Elastodynamics; Discontinuous Galerkin method; Impulsive response; Unconditionally stable methods; Dynamic plate load test; LFWD

1. Introduction

Numerical methods for computing the dynamic response of the ground and soil-structures, such as embankments, bridges and pile foundations, and tunnels, are an indispensable tool for designing and monitoring ground structures. The Finite Element Method (FEM) has been undoubtedly accepted as the most popular, reliable, and successful numerical tool for simulating soil dynamics problems (Anandarajah, 1993; Khoshnoudian and Shahrour, 2002; Kimura and Zhang, 2000; Kuwano et al., 1991; Li and Ugai, 1998; Murakami et al., 2010). Further, environmental impact and safety assessments of the ground vibrations generated by high-speed trains (Ditzel et al., 2001; Krylov, 1996; Schrøder and Dyre, 2008; Takemiya, 2003) and road traffic (Chua et al., 1992; Clemente and Rinaldis, 1998; Clouteau et al., 2001; Maeda et al., 1998) close to residential areas, hospitals, and high-tech industries are of paramount importance. FEM has also been extensively used for investigating the level of vibrations and schemes to reduce them (Hung et al., 2001; Ju, 2009, 2007; Ju et al., 2006; Ju and Lin, 2004). Moreover, in the field of transportation engineering, nondestructive deflection testing, with the Falling Weight Deflectometer (FWD), has become increasingly popular for monitoring the structural health of flexible pavement systems over time (Chai et al., 2015; Elbagalati et al., 2016; Maina et al., 2000). In FWD testing, the recorded

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time histories of surface deflections are used to backcalculate the moduli of pavement layers by employing parameter-identification techniques (Goktepe et al., 2006). Recently, FEM has been employed extensively to simulate FWD testing under realistic conditions and to predict the response of pavement systems brought about by impulsive loading (Loizos and Scarpas¶, 2005; Nazarian and Boddapati, 1995; Picoux et al., 2009).

In the preceding discussion, the most extended approach for solving elastodynamics problems was based on a semi-discretized FEM formulation. In this approach, FEM is first applied to the spatial domain leading to a system of ordinary differential equations (ODE) in the time domain. Subsequently, direct time integration schemes, based on the finite difference method, are employed to solve the resulting system of ODEs. In general, the errors in the finite element simulations of soil-dynamics problems are partially due to the space-time discretization and to the uncertainties involved in the material constitutive relationship and the material parameters. Numerical errors can be reduced by adopting high-order accurate time solvers within FEM framework. Unconditionally stable methods, such as the Newmark-beta method (Newmark, 1959), the HHT–α method (Hilber et al., 1977), the Houbolt method (Houbolt, 1950), and the Wilson–θ method (Wilson et al., 1972), are the most often used dynamic solvers. However, these methods are only second-order accurate. Furthermore, Dahlquist's theorem says that a single-step unconditionally stable time-integration algorithm can be at most second-order accurate (Dahlquist, 1963). High-order accurate time-integration schemes based on finite difference schemes can be employed. However, these schemes are generally conditionally stable and require very small time steps for stability (Hughes, 1983). The requirement of small time steps increases the overall computational cost of large-scale simulations. Therefore, there is still a need for dynamic solvers which are high-order accurate and unconditionally stable, such that large time steps can be used to reduce the computational cost while maintaining the accuracy of the solutions. The aim of the present study is to demonstrate such a scheme.

To achieve high-order accuracy combined with stability, an alternative strategy called the Space-Time Finite Element Method (ST/FEM) was proposed in the past; it is still being actively researched. The concept of finite elements in the time domain was first exploited independently in Argyris and Scharpf (1969) and Fried (1969) using Hamilton's principle. Unfortunately, these formulations were found to be inconsistent, and even inaccurate in some cases, due to the vanishing variation in the displacement at the end points of the time interval. Soon after, Bailey (1982, 1980, 1975) and Simkins (1981) established that instead of Hamilton's principle, *Hamilton's law of varying action*, HLVA, should be used as the starting point for the time-finite element formulations, as it includes the initial conditions implicitly. However, for quite some time, researchers misinterpreted the so-called "trailing terms" of HLVA, and believed that both the displacement field and the velocity field should be continuous in time (Borri et al., 1985b). Due to this misconception, many studies adopted Hermite cubic polynomials as the lowest order of interpolation functions for the displacement field (Baruch and Riff, 1984, 1982; Gellert, 1978; Geradin, 1974; Howard and Penny, 1978; Riff and Baruch, 1984; Simkins, 1981; Sorek and Blech, 1982). Later, Borri and his colleagues (Borri, 1986; Borri et al., 1985a, 1985b) showed that the need to use Hermite polynomials can be avoided by employing Hamilton's weak principle (HWP) in which approximating functions should ensure the continuity of only the displacement field (i.e., C_0 continuity in time), and not that of any of its time-derivatives. More recently, unconditionally stable time-finite element formulations have been derived using the two-field form of HWP instead of the primal form (Aharoni and Bar-Yoseph, 1992; Borri et al., 1991, 1990; Borri and Bottasso, 1993; Mello et al., 1990; Ruge, 1996). For a comprehensive overview and recent advances in the time-finite element formulations based on Hamilton's law, readers are referred to Tamma et al. (2011).

Another approach towards formulating ST/FEM is by weighted residual techniques, such as the continuous Galerkin method (Bajer and Bohatier, 1995; Bajer, 1987), the discontinuous Galerkin method (Hughes and Hulbert, 1988), and the Petrov-Galerkin method (Fung, 1999), while working directly with the differential equations rather than a variational principle. Moreover, ST/FEM derived from the time-discontinuous Galerkin (TDG) method led to the unconditionally stable and high-order accurate ODE solvers (Delfour et al., 1981; Johnson and Pitkäranta, 1986; Lesaint and Raviart, 1974). The discontinuous Galerkin (DG) method was first developed for the neutron-transport equation (Lesaint and Raviart, 1974; Reed and Hill, 1973). Soon after, the TDG method was introduced by Jamet (Bonnerot and Jamet, 1979; Jamet, 1978) for solving the parabolic differential equation with the time-dependent spatial domain. Almost a decade afterwards, Hughes and Hulbert presented the time-discontinuous space-time FEM (TD/ST/FEM) for the fields of elastodynamics and structural dynamics (Hughes and Hulbert, 1988). In this paper, two general formulations of TD/ST/FEM were provided: (i) the single-field formulation, in which the displacement field is the primary unknown, and (ii) the two-field formulation, in which both the displacement field and the velocity field are treated as the primary unknowns. In a latter approach, the trial functions for both fields were continuous in space and discontinuous in time, whereas the test functions were continuous in both space and time (see Section 3). In the two-field formulation, the displacement-velocity compatibility condition and the continuity of both unknown fields in time were satisfied in a weak sense by using some inner products. This is the key element allowing for the generalization of TD/ST/FEM developed for first order hyperbolic equations to the second order hyperbolic equations (Hughes and

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