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On the displacement of a traditional retaining wall when first loaded

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Abstract

When the dimensions of the base of a retaining wall have been selected by, for example, some form of limit state analysis (Butterfield, 2012) an assessment ought also to be made of the displacement of the wall during its working life. This paper presents a simple rational means of doing so, illustrated by but not limited to, loads imposed during a conventional backfilling process. The analysis presented is an application of a vertical-displacement-hardening plasticity model incorporating nested interaction diagrams as yield functions together with a plastic potential and a hardening rule. The hardening rule, a key component of such a model, is assumed to depend only on the vertical centreline load V_0 versus vertical displacement w_0 relationship of the wall base. The model predicts that the horizontal, rotational and vertical displacements will be very simply inter-related; a prediction reinforced by the very satisfactory agreement obtained between it and the results from tests on a model footing supported on dense sand.

Keywords: Design; Footings; Foundations; Plasticity; Retaining walls; Soil/structure interaction

1. Introduction

Displacement prediction of all engineering structures is an important issue, now prominent, in EuroCodes and all serviceability-state considerations. Retaining walls proliferate, yet 'design stage guidance' on assessing their displacement is currently not available.

The objective of the following analysis is to introduce a very simple, analytically well-founded, means of predicting the initial displacement of a gravity retaining wall, such as that illustrated in Fig. 1, when loaded by forces generated during continuous, uniform back-filling. The key results are approximate relationships between the vertical, horizontal and rotational displacements of the wall from predictions of the loads acting upon it using a model with very few parameters. It is therefore presented as a design aid in much the same way that "nested interaction diagrams" for predicting the load capacity of pad foundations were initially conceived (Ticof, 1977). It also provides a basis for displacement predictions due to other loading events throughout the lifetime of the wall. Gravity retaining wall foundations are usually shallow, and can therefore be analysed within the framework of well established strain-hardening plasticity models (also known as "macro-element models"). Such models, applicable to gravity foundations in general, are now widely used in the offshore industry, enabling us to predict both their strength and stiffness (Butterfield, 1978; Gottardi et al., 1999; Houlsby and Cassidy, 2002; Cassidy et al., 2004a,b; Randolph and Gourvenec, 2011).

The basis of the formulation is the use of external resultant forces (V, M, H) and the associated displacements (w, θ, u) of the foundation, considered as generalised

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Notation

a, b, B, h	wall geometry
$d\mathbf{Q}, d\mathbf{q}$	load and displacement increment vectors
D_r	relative density
e	eccentricity of the vertical load
f, g	yield function and plastic potential
H	scalar hardening modulus
K_a	active pressure coefficient
p, q	inclination of the load paths in the (H, V) and $(M/B, V)$ planes
t_h, t_m	parameters of the model
V, M, H	vertical, horizontal and moment load components
V_{max}	bearing capacity for a vertical central load
V^*	peak load
V_0	value of V at tip of yield parabola on the V axis
w, θ, u	vertical displacement, rotation and horizontal displacement
y	height of the fill
β	parameter of the model
γ	soil unit weight

stress-strain variables, with their behaviour governed by a strain-hardening plasticity relationship, from which, for any given load path, the displacements of the foundation can be calculated.

In the specific case discussed in this paper the wall itself is rigid, the displacements are all plastic, the forces and displacements are assumed to increase monotonically but are otherwise time independent. In addition, the forces acting on the wall, the load path, the yield function f and the plastic potential g are both maximally simplified and assumed to be known. An essentially identical analysis can be developed whatever form f and g might take.

2. A simplified vertical-displacement-hardening plasticity model

The essence of the model is that a general load increment vector $d\mathbf{Q} = \{dV, dM, dH\}$, acting centrally on the base at O in Fig. 1, is related to the associated displacement increment vector $d\mathbf{q} = \{dw, d\theta, du\}$ by the classical strain-hardening plasticity relationship

$$d\mathbf{q} = -\frac{1}{\mathbf{H}} \cdot \mathbf{C} \cdot d\mathbf{Q} \tag{1}$$

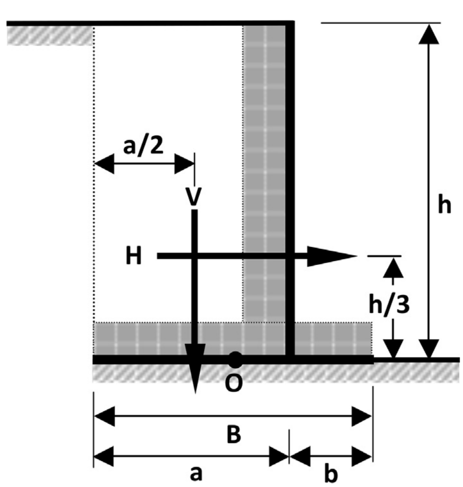

Fig. 1. Typical loading on a retaining wall.

where H is a scalar hardening modulus and **C** a (3 × 3) compliance matrix the terms in which are products of partial differentials of a yield function f and a plastic potential g, such as, in our case, $(\partial f/\partial V),(\partial g/\partial H)$ etc.

The equation of a relevant, experimentally established, yield function, in terms of (V, M/B, H) is (Butterfield and Gottardi, 1994)

$$f = \left(\frac{H}{t_h}\right)^2 + 2\beta\left(\frac{HM}{Bt_h t_m}\right) + \left(\frac{M}{Bt_m}\right)^2 - \left\{V\left[1-\left(\frac{V}{V_0}\right)\right]\right\}^2 = 0 \tag{2}$$

in which the two parameters (t_h, t_m) have values close to (1/2, 2/5) = (0.50, 0.40) for near-surface footings. In addition to Gottardi's numerous dense-sand results closely similar values have been obtained for other types of soils and foundations (Byrne and Houlsby, 1999; Houlsby and Cassidy, 2002; Cassidy et al., 2004a,b; Gottardi et al., 2005; Villalobos et al., 2009; Govoni et al., 2011). Consequently these values of (t_h, t_m) are likely to provide reasonable design guidance for the base of a retaining wall.

For any value of Vo, f plots as a three-dimensional, cigar-shaped surface rotated by $\beta = 0.22$ rad around the V axis. When Vo = Vmax the expanding yield locus becomes identical to the failure envelope for the surface foundation used in the limit-state design process described in Butterfield (2012). Fig. 2 shows a planar, nested set of geometrically similar yield loci in the (V, H) plane. Each member of the set is a parabola, defined by the value of V = Vo at its tip, with the equation,

$$\frac{H}{V} = t_h\left(1-\frac{V}{V_0}\right);\frac{V}{V_0} = \left(1-\frac{H}{Vt_h}\right) \tag{3}$$

Typical H/V = p = constant load paths are also shown in Fig. 2; most of the subsequent calculations are related to this figure. It is also well established (Gottardi et al., 1999; Cassidy et al., 2004a,b) that, for a soil-supported foundation, the displacement increment vectors d**q** along any 'radial' load path in the (V, H) plane are not orthogonal to f = 0 (i.e. flow is non-associated), but are closely parallel to each other until the footing approaches failure (Gottardi