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Staircase predictor models for reliability and risk analysis

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ABSTRACT

This paper proposes a technique for constructing computational models describing the distribution of a continuous output variable given input-output data. These models are called Random Predictor Models (RPMs) because the predicted output corresponding to any given input is a random variable. We focus on RPMs having a bounded support set and prescribed values for the first four moments. This prescription, to be realized by staircase variables, enables modeling skewed and multimodal phenomena distributed over an input-dependent interval. Responses with such complex features often arise in structural dynamics. As an example we consider the reliability analysis of an aeroelastic airfoil subject to flutter instability whose data is corrupted by model-form uncertainty and measurement noise. Furthermore, we propose a risk analysis methodology to trade-off performance against reliability. This example demonstrates that substantial performance improvements are obtained by (taking the risk of) ignoring a small percentage of the predicted responses.

1. Introduction

Metamodeling [1] refers to the process of creating a mathematical representation of a phenomenon based on input-output data. Metamodels can be parametric, e.g., polynomial response surfaces, polynomial chaos expansions, or nonparametric, e.g., Gaussian Process (GP) models, smoothing spline models, multiplicative regression, Kernel and additive model regressions. In nonparametric regression the predictor does not take a predetermined form but is constructed according to information derived from the data. Nonparametric regression requires larger sample sizes than parametric regression because the model structure as well as its hyper-parameters must be inferred. The metamodels proposed below are nonparametric.

A GP model [2] is a Random Predictor Model (RPM) in which the predicted output is a normal random variable having input-dependent mean and covariance functions. GP models are a powerful Bayesian tool for nonlinear regression, function approximation, and predictive density estimation. GP models describe the observations as the sum of an unknown latent function plus a Gaussian noise. Unlike other regression methods, GPs proceed in a Bayesian fashion to infer the posterior distribution of the unknown function through the likelihood and a prior distribution placed over the unknown function. GPs usually employ a small number of hyper-parameters which are tuned by optimization. GP models are widely used due to their ability to characterize complex functional relationships between inputs and output, and to account for the effects of making predictions away from the range of the data. All these advantages come with a price: the computational requirements scale cubically with the number of data points, thereby necessitating approximations and data reduction when the dataset is large. Furthermore, GPs require making strong assumptions on the perceived noise, whose power is often considered constant throughout the input space (i.e., homoscedastic), and on the covariance function of the prior, which is typically modeled as depending only on the difference between input values. More importantly, the intrinsic structure of GP models restricts their applicability to phenomena trending strongly towards bell-shaped, symmetric and unimodal distributions. This is not the case for Gaussian Mixture (GM) Models [3,4] and Mixture of Gaussian Process Experts [5], which enable generating smooth multimodal predictors by combining multiple stationary GPs. However, the learning and inference operations within such models require simulating the corresponding process using Markov Chain Monte Carlo making them computationally expensive to calculate and tune. Furthermore, strong data clustering often lead to overly complex predictors having unnecessary components.

This paper proposes a class of RPMs having the versatility to describe complex responses typical of engineering systems. Measurement noise, model-form uncertainty, and parametric uncertainty often lead to aleatory responses whose features vary strongly with the input variables. For instance, a flexible structure subject to aleatory uncertainty in its parameters and initial conditions often renders non-gaussian time responses. This is also the case of the frequency response function obtained by experimental modal analysis, in which variations in the system's parameters, boundary conditions, and sensor dynamics yield complex distributions for both the magnitude and phase. The

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complexity of the response often stems from its nonlinear dependency on the system parameters. This is the case for most linear and nonlinear dynamical systems. The latter case is exemplified by changes in the system response caused by bifurcations, e.g., limit cycle oscillations, buckling phenomena, etc. The proposed RPMs can describe skewed and multimodal responses by manipulating an outer bound to the range of the response and its first four moments. Making the prediction match the observations by adjusting hyper-parameters is a long standing approach in reliability-based design optimization, moment matching algorithms, and backward propagation of variance [6–8]. The predictors proposed herein are of the moment-matching type.

This paper is organized as follows. Section 2 presents the problem statement and main goals of this article. This is followed by a brief introduction to the staircase variables at the core of the predictor. Sections 4 and 5 provide a means to calculate interval and random predictor models based on the available data. Examples describing the application of the approach to an easily reproducible toy example, its performance relative to competing alternatives, and its usage in a structural dynamics application are then provided. Finally, a discussion section and a few concluding remarks close the paper.

2. Problem statement

A Data Generating Mechanism (DGM) is postulated to act on a vector of input variables, $x \in \mathbb{R}^{n_x}$, to produce one or more outputs, . In this article the focus will be on the single-output ($n_y = 1$) multi-input ($n_x \geqslant 1$) case. The dependency of the output on the input is arbitrary. This covers the case in which y is a function of x with all components of x available (so there is only one output value for each available input), the case in which y is a function of x but not all components of x are available (so there might be infinitely many outputs for each measured input[1]), and the case in which y is an arbitrary random process of x. Assume that N Independent and Identically Distributed (IID) input-output pairs are obtained from a stationary DGM, and denote by $\mathbb{D} = \{x^{(i)}, y^{(i)}\}$, $i = 1, \ldots N$, the corresponding data sequence. The main objective of this article is to generate a computational model of a DGM based on its observations $\mathbb{D}$. Two types of predictors will be developed. An Interval Predictor Model (IPM) [9,10] yields a bounded interval of output values at any value of the input. The desired IPM is a narrow interval that not only contains all the data but it will also contain future data with high probability. Conversely, a Random Predictor Model (RPM) yields a random variable at any value of the input. The desired RPM accurately describes the probability distribution governing the DGM.

3. Preliminaries

Consider the continuous random variable z with support set $\Delta_z = [z_{\min}, z_{\max}]$, Probability Density Function (PDF) $f_z\colon \Delta_z \subset \mathbb{R} \to \mathbb{R}^+$, and Cumulative Distribution Function (CDF) $F_z\colon \Delta_z \to [0, 1]$. Denote by m_r the r-th central moment of z, which is defined as

$$m_r = \int_{\Delta_z} (z-\mu)^r f_z(z) dz, \quad r = 0, 1, 2, \ldots \tag{1}$$

where μ is the expected value of z. Note that $m_0 = 1$, $m_1 = 0$, m_2 is the variance, m_3 is the third-order central moment, and m_4 is the fourth-order central moment.

The random variables at the core of the proposed RPMs are constrained to have a bounded support set and given values for μ, m_2, m_3, and m_4. The bounded support constraint is given by $\Delta_z \subseteq \Omega_z$, where $\Omega_z = [\underline{z}, \overline{z}]$, with $\overline{z} \geqslant \underline{z}$. The moment constraints are the equality constraints in (1). The parameters of these constraints will be grouped into the variable $\theta_z \in \mathbb{R}^6$ given by

$$\theta_z = [\ \underline{z}, \overline{z}, \mu, m_2, m_3, m_4\ ]. \tag{2}$$

Any random variable z having a support set contained by $[\underline{z}, \overline{z}]$ with moments μ, m_2, m_3, and m_4 must satisfy the feasibility conditions $g(\theta_z) \leq 0$ given in [11]. The realizations of θ satisfying these conditions constitute the θ-feasible domain, Θ, defined as

$$\Theta = \{\theta\colon g(\theta) \leqslant 0\}. \tag{3}$$

Determining membership in Θ is a distribution-free assessment applicable to possibly infinitely many random variables. Staircase random variables [11] are able to realize most of Θ. They are called staircase because the PDF of its members is piecewise constant over bins of equal width. Staircase variables are calculated by solving the optimization program

$$\min_{\ell \geqslant 0} \{J(\theta, n_b)\colon A(\theta, n_b)\ell = b(\theta),\ \theta \in \Theta\}, \tag{4}$$

where J is the cost function, n_b is the number of bins partitioning Ω_z, ℓ are the PDF values at the bin centers, and $A\ell = b$ are moment matching constraints. This optimization program is convex when the cost J is a convex function. This is the case for several optimality criteria, including maximal-entropy E, and maximum-likelihood L (see Appendix for details). Hereafter, staircase variables will be denoted as

$$z \sim S_z(\theta_z, n_b, J). \tag{5}$$

The number of bins n_b determines the staircase feasibility of a θ-feasible point. The staircase-feasible domain is defined as

$$\mathscr{S}(n_b) = \{\theta \in \Theta\colon \exists\ \ \ell \geqslant 0 | A(\theta, n_b)\ell = b(\theta)\}. \tag{6}$$

Hence, the set $\mathscr{S}(n_b)$ is comprised of all realizations of θ for which an staircase variable having n_b bins exists. Increments in n_b rapidly reduce the size of the offset between Θ and $\mathscr{S}(n_b)$. The dependency of the staircase PDF on the number of bins n_b is illustrated in Fig. 1. Note that the underlying shape of the PDFs is fairly insensitive to the number of bins. However, S_z becomes infeasible when $n_b < 6$. The influence of the number of bins is not critical. Numerical experiments indicate that a density of 100 bins per unit of length yield staircase variables that match most of Θ.

Staircase variables have the ability to represent a wide range of density shapes and the low-computational cost required to efficiently perform many uncertainty quantification tasks. The staircase RPMs proposed below will be prescribed by making each of the six input arguments of θ in (2) a function of the input x. The reminder of this paper focuses on how to set such functions given $\mathbb{D}$.

4. Interval predictor models

This section presents a means to calculate an IPM that tightly encloses the data. Such an IPM will later be used to prescribe the support set of a staircase RPM. Additional information is available in [10].

An IPM assigns to each instance vector $x \in X \subseteq \mathbb{R}^{n_x}$ a corresponding outcome interval in $Y \subseteq \mathbb{R}$. That is, an IPM is a set-valued map, $I_y\colon x \to I_y(x) \subseteq Y$, where $I_y(x)$ is the prediction interval. Depending on context, the term IPM will refer to either the function I_y or its graph $\{(x, y)\colon x \in X, y \in I_y(x)\}$ in $X \times Y$. A nonparametric IPM is given by

$$I_y(x) = \{[\underline{y}(x),\ \overline{y}(x)],\ \overline{y}(x) \geqslant \underline{y}(x)\}. \tag{7}$$

where the functions $\underline{y}(x)$ and $\overline{y}(x)$ are the lower and upper boundaries of the IPM respectively. A parametric IPM is obtained by associating to each $x \in X$ the set of outputs y that result from evaluating the parametric model $y = M(x, p)$ at all values of p in the set P, so $I_y(x, P) = \{y = M(x, p),\ p \in P\}$. Attention will be limited to the case in

[1] Consider the two-input single-output DGM $y(x_1, x_2)$, where y is a deterministic function of its two inputs. When x_1 is the only controllable/measurable input, observations of the output corresponding to a fixed value of x_1 will vary in a range. This variation is caused by unknown variations of x_2. The observed output variation is not caused by noise per se, but its characterization as noise will almost always require it to be modeled as heteroscedastic. As such, this is a situation in which the DGM is deterministic but model-form uncertainty yields seemingly random data.

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