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Dimension adaptive finite difference decomposition using multiple sparse grids for stochastic computation

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ABSTRACT

The present study aims to investigate uncertainty quantification followed by reliability analysis of structure with homogeneous non-normal random fields. In stochastic finite element formulation, these continuous fields are discretized by different methods (e.g. Karhunen-Loève Expansion) which transformed it into a set of random variables. However, this discretization often leads to large number of random variables, especially for multiple random fields. With this in view, two different meta-model based approaches are presented in this study using high dimensional model representation (HDMR) for efficient stochastic computation. First, an adaptive multiple finite difference HDMR (AMFD-HDMR) is proposed that decomposes the original performance function into summands of smaller dimensions. These subfunctions are modeled by polynomial chaos expansion (PCE) using moving least square technique which utilizes the benefits of orthogonality of the basis functions and provides adaptive interpolation between the support points. These support points are generated in a sparse grid framework based on the hierarchial tensor products of the sub-grids and the appropriate statistical properties. An iterative scheme is developed with the aim to create new support points in the desired locations such as most probable failure point and/or maxima/minima. Later, a dimension adaptive multiple finite difference HDMR (*d*AMFD-HDMR) is proposed utilizing sensitivity analysis to further improve the efficiency and accuracy. In the second proposal, an intermittent HDMR formulation is suggested based on the individual and mutual contributions of the significant dimensions. Once the meta-model is built, Monte Carlo simulation is performed over it, thus bypassing the time exhaustive computation of the original performance function. Numerical studies are carried out using composite plate to prove the merits of the proposed algorithms compared to other methods available in the literature.

1. Introduction

Monte Carlo simulation (MCS) is widely preferred for stochastic computation due to its robustness and ability to deal with complex limit states. However, it often demands exhaustive computational cost especially when the performance function is modeled in finite element framework. Optimizing this cost using effective computational methods have remained an open field of research in the recent past. In this context, Ghanem and Spanos [1] developed spectral stochastic finite element method (SSFEM) which considered uncertainty as an additional dimension of the system using polynomial chaos expansion (PCE) and Karhunen-Loève Expansion (KLE). Uncertainties were modeled using random field approach which is more generalized representation of the uncertain perturbations than random variable approach as it either under or over-estimate the moments [2,3]. However, SSFEM suffered inaccuracy [4] as PCE with Galerkin approach inadequately modeled the failure region for low probability while increasing the order of PCE required high computation time [5]. A generalized version of SSFEM was proposed by Xiu and Karniadakis [6] with different orthogonal polynomials from Askey scheme such as Laguerre, Jacobi, Legendre etc. for accelerated convergence [7]. Ghanem et al. [8] improved the efficiency of SSFEM by combining it with PCE through dimension reduction. Here, the term *dimension* refers to the number of variables and same is referred throughout this paper. The idea of dimension reduction arose through the observation that eigenvalues from KLE decay for short correlation length beyond certain terms. Maute et al. [9] also proposed reduced order model for PCE which required multiple updating of the reduced basis matrix based on perturbations of the random variables. Furthermore, the application of SSFEM for different complex problems with random field in various domains like fluid mechanics [10], soil mechanics [11,5] and so on have been extensively studied in the recent past.

In particular, it can be noticed that SSFEM required modification of the governing equation to incorporate the uncertainty. This task could

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be difficult for system with complex governing equation which is often dealt in civil and mechanical engineering. Hence, one can prefer non-intrusive formulation [12] using collocation points (i.e. Gauss quadrature points) and Latin hypercube sampling (LHS) where the system is regarded as *black-box*. Following it, stochastic response surface method (SRSM) [13,14], PCE with bi-orthogonal polynomials [15] and response surface methodology (RSM) [16,17] for random field problems were developed. Ahmed and Soubra [18] extended the application of subset simulation for reliability analysis using SRSM on ad hoc basis for approximate probability of failure estimation. Recently, Rathi et al. [19] proposed a sequential SRSM using moving least square (MLS) based PCE with Hermite polynomial basis for reliability analysis. It showed the adaptiveness of MLS based PCE formulation compared to collocation based PCE. These numerical approaches provide wide applications as it seeks the values of the original performance function at some known locations which are often called *support points* or *experimental points* or *training points*. The number of these support points are strongly related to the dimension of the problem.

Although intrusive and non-intrusive methods are effective but there exists certain issues with efficiency and accuracy for problems having large dimension, nonlinearity etc. Researchers have developed various techniques and algorithms to improve it by either reducing the terms or dimension. Among them, model decomposition is used to represent high dimensional performance function into multiple component functions of low dimensions using analysis of variance (ANOVA) [20–23] which is often called high dimensional model representation (HDMR) [24]. This expansion technique has been suggested using random sampling [25,26] as well as deterministic sampling [27]. Chowdhury and Adhikari [27] proposed 1st and 2nd order HDMR with regular polynomial basis up to degree 2 for uncertainty quantification of random field problems discretized by KLE up to 10 terms. Extending the application of HDMR, Chen et al. [28] used 1st order approximation [22] for robust topology optimization with a random field discretized by reduced KLE. Using ANOVA based decomposition technique, Chakraborty and Chowdhury [29,30] proposed polynomial correlated function evaluation (PCFE) with orthogonal basis. Blatman and Sudret [31–34] reduced the number of unknown coefficients using sparse polynomial chaos approximation in a regression framework. Insignificant terms were dropped based on their influence on R^2 value (i.e. coefficient of determination). Application of sparse PCE has been studied for time-dependent reliability analysis with principal component analysis to reduce the number of time steps (i.e. dimension of the time-variant problem) [35]. Alternatively for dimension reduction, Pan and Dias [36] applied sliced inverse regression in conjunction with sparse PCE for random fields discretized by KLE. Tipireddy and Ghanem [37,38] proposed adaptive basis based PCE for intrusive as well as non-intrusive formulations using projections. Sasikumar et al. [39] studied the effect of random field to evaluate the reliability of composite plate using optimal linear expansion (OLE). Properties of the layered composite plate were defined by non-Gaussian distributions based on the experimental results [40]. Application of OLE reduced the number of discretization terms through construction of coarse random field mesh. They also applied PCE based SSFEM to study the propagation of uncertainty in the composite plate [41]. It helped in overcoming the issue of singularity in OLE where inversion of correlation matrix becomes difficult for large correlation length. Mahjudin et al. [42] proposed a semi-analytical approach to determine the statistical moments of plates using certain generalized stresses method. However, the method is limited to thin-walled structures and yield significant error for random field of short correlation length. Although the method is non-intrusive but it doesn't treat the problem as black-box as it requires certain information like displacement field, type of element etc. which restricts its application to complex FEM problems. Bensi et al. [43] proposed an advanced discretization of Gaussian random field. It was done using the transformation matrix updated in Bayesian framework. The insignificant terms (i.e. links and nodes) required for updation of the matrix were truncated based on constrained optimization to minimize error in the correlation matrix.

2. Problem formulation

The numerical approaches described in the above literature effort to build high fidelity models with minimal computational cost which is proportional to the number of original function calls. In order to do so, theses methods (a) limit the number of independent variables through discretization of random field, (b) omit or modify certain terms in the polynomial series and (c) require information of the finite element (FE) model to determine the variability with adequate accuracy and efficiency. Construction of such meta-models require support points generated from either random sampling (i.e. LHS, Sobol' sequence) or deterministic sampling (i.e. collocation points, sparse grid). These techniques employ a single generation of the support points whereas the present study prescribes multiple sequential generation. Optimization process is followed to find specific location(s) like most probable design point, maxima, minima etc. as desired by the designer. However in reliability analysis, design point based approximation of limit state performs poorly for large dimensions (say >20) [44]. This is due to the fact that in high dimensional problems, multiple regions contribute to failure which are otherwise overlooked by approximate gradient based methods. The present study extends this limitation of design point based approximation by implying sequential and distribution adaptive generation of the support points. For efficiency, sparse grid scheme is adopted with the optimization process, performed over the meta-model, aiding in the sequential generation within the desired domain. The meta-model is formed iteratively using multiple HDMRs which adaptively fit the original surface. Here, we proposes two new methods to perform stochastic computation for high dimensional problem e.g. structure with random fields. First method employs MLS based PCE approximation under the framework of multiple HDMRs with sequential sampling using sparse grid scheme. The second approach introduces ad hoc sensitivity analysis for dimension reduction over the previous approach. It can be noticed that the second approach reduces the dimensions based on their variability, unlike some literatures [13,14,27,37] which directly curtail the terms in the random field discretization. Hence, extending its application for problems with random fields having short correlation length. With this in view, the present study focuses on:

- Efficient uncertainty quantification and reliability analysis using PCE based HDMR formulation with sequential generation of support points following sparse grid scheme.
- Development of an adaptive meta-model with effective dimension reduction for high dimensional problems with random field.
- Improving the efficacy of design point based approximation for high dimensional problems using multiple HDMRs.

Additionally, this study compares the performance of RS-HDMR [25] with KLE for stochastic computation of composite plate subjected to non-Gaussian random fields. Furthermore, the proposed methods are also compared with other methods available in the literature to elucidate their merits.