ARTICLE IN PRESS

Structural Safety xxx (2017) xxx–xxx

Contents lists available at ScienceDirect

Structural Safety

journal homepage: www.elsevier.com/locate/strusafe

Low-cost finite element method-based reliability analysis using adjusted control variate technique

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ARTICLE INFO

Article history:
Received 20 April 2017
Received in revised form 1 November 2017
Accepted 27 November 2017
Available online xxxx

Keywords:
Finite element method
Mesh density
Reliability analysis
Control variate technique

ABSTRACT

Reliability analysis is used to evaluate the safety of engineering structures subject to uncertainties. Finite element method (FEM) is a popular engineering tool used to evaluate the reliability of complex engineering structures. In general, FEM–based reliability analysis of engineering structures is influenced by the mesh density of the model and the accuracy of the results requires the use of a very fine mesh density in the analysis. However, it is often impractical for reliability analysis complex structures, especially those with low failure probabilities. Hence, a new method is proposed to address this issue, which provides an accurate estimate of the failure probability at low computational cost. In this method, the control variate technique is used in conjunction with the FEM-based reliability analysis, where the failure probability integral is broken down into two separate integral terms. The first term provides a low-cost estimate of the failure probability using a model with coarse mesh density, whereas the second term regulates the failure probability based on fewer finite element analyses with fine mesh density. The adjusted correction factors are also presented in this paper in order to improve the efficiency of the proposed approach. The proposed approach is used to estimate the reliability index of four engineering structures and the results show that the method is efficient and practical for FEM-based reliability analysis of engineering structures.

1. Introduction

A fundamental problem in structural reliability theory is to compute the failure probability (P_f), which is a multifold probability integral defined as:

$$P_f = Prob[g(x) \leqslant 0] = \int_{g(x)\leqslant 0} f(x)dx, \quad (1)$$

where x is a vector of random variables representing uncertain structural quantities. The functions $g(x)$ and $f(x)$ denote the limit state function and the joint probability density function (PDF) of x, respectively.

In most engineering applications, the multifold probability integral given by Eq. (1) is difficult to compute because it involves multi-dimensional integration, where the dimension equals to the number of basic random variables.

Various analytical and simulation methods have been developed over the years to solve the integral above. First order reliability methods (FORMs) are typically used to estimate the failure probability without incurring long computational processing time [1–4]. However, the main disadvantage of these methods is that they often do not yield accurate results for cases involving non-normal distributions, limit state functions that are highly nonlinear, multiple basic variables, and complex failure surfaces [5]. For this reason, a number of simulation methods have been developed to compute the failure probability with high accuracy [5–7]. One of these methods is Monte Carlo simulation (MCS), which involves generating random samples based on the mean value of the variables [6].

For small failure probabilities, the MCS method is a rather time-consuming approach due to the large number of samples required [6–8]. This disadvantage may be eliminated by using an instrumental PDF, $h(x)$, to generate more samples within the failure region:

$$P_f = \int_{g(x)\leqslant 0} \left(\frac{f_x(x)}{h_x(x)}\right) h_x(x)dx, \quad (2)$$

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https://doi.org/10.1016/j.strusafe.2017.11.005
0167-4730/

Please cite this article in press as: Rashki M et al. Low-cost finite element method-based reliability analysis using adjusted control variate technique. Struct Saf (2017), https://doi.org/10.1016/j.strusafe.2017.11.005

This method is known as importance sampling (IS). The importance sampling estimator is given by [9,10]:

$$P_f = \frac{1}{N}\sum_{i=1}^{N} I[g(x_i)]\frac{f(x_i)}{h(x_i)}, \tag{3}$$

The weighted average simulation method (WASM) is also an efficient simulation method to compute the failure probability and determine the most probable point (MPP) [2,6–8]. In this method, random numbers are first generated based on the initial assumption of the failure probability. Next, a weight index is assigned to the generated samples based on their competency and the failure probability is estimated using the following equation:

$$P_f = \frac{\sum_{i=1}^{N} I[g(x_i)](\prod_{j=1}^{s} f_j(i))}{\sum_{i=1}^{N}(\prod_{j=1}^{s} f_j(i))}, \tag{4}$$

In addition, other methods such as line sampling (LS), subset simulation (SS), metamodel line sampling, and unbiased metamodel method have been developed to overcome the limitations of MCS for various engineering problems [11–14].

However, these methods may not be feasible in practice when the performance function needs to be solved using a time-consuming approach such as finite element method (FEM).

In deterministic analyses, mesh convergence analysis [15,16] and grid convergence index (GCI) [17–20] are used to select the suitable finite element (FE) model and the simulation results are compared with those obtained from analytical functions or experiments [21–24]. Once mesh convergence is achieved, the differences in the FE results obtained from different mesh densities will be small and these small errors are considered as acceptable. Several researchers have assessed the effects of mesh density in deterministic analyses [25,26]. For example, Waide et al. [27] investigated the load transfer characteristics of two types of cemented hip replacements with fibrous tissue layer using FEM and they compared the results with those obtained from experiments. The results showed that the maximum difference between the FE and experimental results was 15%. In addition, one study on spinal segments showed that the difference in the FE results was less than 5% once mesh convergence was attained, which the researchers perceived as adequate [28].

However, there are very few studies focused on the selection of a suitable FE model for probabilistic reliability analysis [29].

This study shows that the errors considered as acceptable in deterministic analysis (errors arising from inadequate mesh density) have a significant effect on the evaluating the safety of engineering structures and very fine mesh densities are required to estimate the failure probability with reasonable accuracy. Owing to the fact that it is impractical and time-consuming to use models with very fine mesh densities in reliability analysis, a new FEM-based reliability analysis method is proposed in this study to compute the reliability index in a simple, efficient manner with a high degree of accuracy and low computational cost.

2. Development of the adjusted control variate technique (ACVAT) for FEM-based reliability analysis of engineering structures

When the performance evaluation of an engineering structure requires the use of FEM, Eq. (1) which is used to compute the failure probability can be written as:

$$P_f = \int_{g\leqslant 0} f(x)dx \cong \int_{G^{FEA}\leqslant 0} f(x)dx, \tag{5}$$

where $G^{FEA} \leqslant 0$ is the failure region. The performance of the structure in this domain is evaluated by FEM.

Even though it is possible to determine errors due to inadequacy of mesh density in deterministic analysis, it is challenging to determine errors inherent in FEM-based reliability analysis. Thus, the control variate technique (CVT) is adopted in this study to tackle this issue.

Suppose that the objective is to estimate the following failure probability integral:

$$\mathbb{E}(p) = \int p(x)f(x)dx, \tag{6}$$

where $p(x)$ is the function of interest and $f(x)$ is the PDF of the input x. When the function $p(x)$ is not known or complex, estimation of the failure probability integral becomes difficult. Hence, in the CVT, it is assumed that there is another function $g(x)$, which is correlated with $p(x)$ with a known mean. Hence, Eq. (6) can be approximated as [30]:

$$\mathbb{E}(p) = \int g(x)f(x)dx + \int (p(x) - g(x))f(x)dx. \tag{7}$$

In this formulation, $g(x)$ is known as the control variate for $p(x)$. Since the mean of the first term is known (or estimating its expectation is easier than $p(x)$), the method transfers the difficulty of the estimation to the second term. Indeed, $p(x)$ effects on the total estimation are reduced.

For FEM-based reliability problems, Eq. (1) can be written as:

$$P_f = \int_{g\leqslant 0} f(x)dx \cong \int_{-\infty}^{+\infty} p\left(G_{fine}^{FEA}\right)f(x)dx, \tag{8}$$

where

$$p\left(G_{fine}^{FEA}\right) = \begin{cases} 1, & G_{fine}^{FEA} \leqslant 0 \\ 0, & G_{fine}^{FEA} > 0 \end{cases} \tag{9}$$

In this equation, G_{fine}^{FEA} represents the performance function evaluated by the FE model with a very fine mesh density. Solving reliability problems with this specification is impractical for complex engineering problems. Hence, in the proposed approach (i.e., ACVAT), the results obtained from the FE model with coarse mesh density are used as the control variates of the FE model with fine mesh density. Hence, the failure probability integral given by Eq. (7) can be rewritten as:

$$P_f = \int g\left(G_{coarse}^{FEA}\right)f(x)dx + \int \left(p\left(G_{fine}^{FEA}\right) - g\left(G_{coarse}^{FEA}\right)\right)f(x)dx,$$
$$p\left(G_{coarse}^{FEA}\right) = \begin{cases} 1, & G_{coarse}^{FEA} \leqslant 0 \\ 0, & G_{coarse}^{FEA} > 0 \end{cases} \tag{10}$$

where G_{Coarse}^{FEA} represents the performance function evaluated by the FE model with coarse mesh density. It shall be noted that $g\left(G_{Coarse}^{FEA}\right)$ is the control variate of $p\left(G_{fine}^{FEA}\right)$. Hence, Eq. (10) is rewritten as:

$$P_f = \int g(G_{coarse}^{FEA})f(x)dx + \frac{\sum\left(p(G_{fine}^{FEA}) - g(G_{coarse}^{FEA})\right)}{N}, \tag{11}$$

where the sampling for estimation of the second term is performed based on $f(x)$ and N represents the sample size. The first term is the failure probability of the given problem, which is solved using the FE model with coarse mesh density, i.e., $\int g\left(G_{Coarse}^{FEA}\right)f(x)dx = \mathbb{E}(g\left(G_{Coarse}^{FEA}\right)) = P_f^{Coarse}$. Estimating the first term requires lower

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