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# An efficient global reliability sensitivity analysis algorithm based on classification of model output and subset simulation

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ABSTRACT

The global reliability sensitivity analysis measures the effect of each model input variable on the failure probability, which is very useful for reliability-based optimization design. The aim of this paper is to propose an alternative method to estimate the global reliability sensitivity indices by one group of model input–output samples. Firstly, Bayes formula is used to convert the original expression of global reliability sensitivity index into an equivalent form where only the unconditional failure probability and the failure-conditional probability density function (PDF) of each model input variable are required. All global reliability sensitivity indices can be simultaneously estimated by this new equivalent form, and the computational cost of the process is independent of the dimensionality of model input variables. Secondly, to improve the efficiency of sampling which aims at calculating the unconditional failure probability and estimating the failure-conditional PDF of every model input simultaneously, subset simulation method is extended to achieve these two aims. In the proposed procedure, subset simulation is used to estimate the unconditional failure probability, and Metropolis-Hastings algorithm is employed to convert the samples in failure domain from the current PDF in subset simulation to the PDF corresponding to the original PDF of model inputs for estimating the failure-conditional PDF of each model input variable. Thirdly, Edgeworth expansion is employed to approximate the failure-conditional PDF of each model input variable. Finally, the global reliability sensitivity index can be easily computed as byproducts using the unconditional failure probability and the failure-conditional PDF of each model input in failure probability analysis, and this process does not need any extra model evaluations after the unconditional failure probability analysis is completed by subset simulation. A headless rivet model, a roof truss structure and a composite cantilever beam structure are analyzed, and the results demonstrate the effectiveness of the proposed method in global reliability sensitivity analysis.

## 1. Introduction

Reliability analysis aims at measuring the ability that a system or a component achieves its intended performance without failures by taking uncertainties of model input variables into account. During the past several decades, various approximate methods have been developed to estimate the failure probability. These methods are mainly divided into three categories. The first category is the moment-based method such as the first-order reliability method [1,2], the second-order reliability method [3,4], the first-order third-moment method [5] and the fourth-moment method [6–9]. The second category is the sampling-based methods such as the Monte Carlo simulation (MCS), the importance sampling method [10–13], the subset simulation method [14–17], the line sampling method [18,19], the directional simulation [20], etc. The third category is the metamodel based method [21–26] such as AK-MCS [21], AK-IS [22], meta AK-IS$^2$ [23], etc.

To allocate resources to inputs to reduce the failure probability of a system or a component effectively, reliability sensitivity analysis [27] is very effective especially for reliability-based design optimization. Reliability sensitivity analysis is divided into two categories, i.e., the local reliability sensitivity analysis [28] and the global reliability sensitivity analysis [29–32]. Local reliability sensitivity analysis is based on the estimation of partial derivative where the model inputs are fixed at their nominal values. Global reliability analysis aims at quantifying the sensitivity information of model inputs to the failure probability of model output by fixing the inputs over their whole distribution ranges. To rationally measure the global reliability sensitivity of each model input, Cui et al. [29] proposed a moment-independent global reliability sensitivity index which is defined as the mean absolute difference between the unconditional failure probability and the conditional failure

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probability. Li et al. [30] proved that this index is consistent with the Sobol' index [33–35] when squared operation is used where the decomposed function is changed from the original model output to the indicator function of failure domain. Wei et al. [31] standardized this index and proposed three efficient single-loop methods to estimate this index by inducting the MCS, importance sampling method and truncated importance sampling method.

The aim of this paper is to propose an alternative method to estimate the global reliability sensitivity index proposed in Refs. [29–31] and the proposed alternative sampling-based method is only based on one group of input-output samples. The surrogate method is efficient for analyzing the finite element, but this type of surrogate method needs a post-processing computational cost to evaluate the global reliability sensitivity indices. Although the post-processing computational cost is usually ignored because it is relatively smaller than that of evaluating the limit state function, it dose still exist. Therefore, an efficient post-processing global reliability sensitivity analysis method can enhance the efficiency of surrogate methods effectively.

Firstly, Bayes formula [36] is employed to construct the relationship between the unconditional failure probability and the conditional failure probability. Based on this relationship, a new equivalent formula based on the classification of model output, i.e., failure group and safe group, is proposed to estimate the global reliability sensitivity indice where only the unconditional failure probability and the failure-conditional probability density function (PDF) of each model input are required. To estimate the new equivalent formula, only one group of model input-output samples is required so that all indices can be estimated simultaneously without any extra model evaluations.

Secondly, the samples involved in reliability analysis are reused to estimate the failure-conditional PDF of each model input. MCS can be chosen, but its efficiency is low for small failure probability and a small number of samples are dropped in failure domain so that the failure-conditional PDF is difficult to estimate accurately. Compared with MCS, subset simulation can drop more samples in failure domain due to expressing the failure probability as a product of a sequence of conditional failure probabilities.

Thirdly, by reusing the samples generated by subset simulation for estimating the unconditional failure probability, Metropolis-Hastings algorithm is employed to transform the failure samples generated by subset simulation into the relevant failure samples following the original failure-conditional PDF. By use of these failure samples following the original failure-conditional PDF, the failure-conditional PDF of each model input can be estimated. To estimate the failure-conditional PDF, Edgeworth expansion [37] is employed in this paper where only the first four-order failure-conditional moments are required which can be estimated by the transformed failure samples under the failure-conditional original PDF.

From the above ideas, global reliability sensitivity indices of all the input variables can be estimated by one group of input-output samples which is also used in unconditional failure probability estimation. Due to the importance sampling based subset simulation method being employed in this paper to estimate the proposed equivalent formula of global reliability sensitivity indices and the design point-based importance sampling method may be inefficient in high dimensional space [12,13], the problems with less than ten random inputs are analyzed in Case study Section to demonstrate the efficiency of the proposed method for problems with low number of random inputs generally less than ten.

The rest of this paper is organized as follows. Section 2 briefly reviews the definition of the global reliability sensitivity analysis. Section 3 introduces the new global reliability sensitivity analysis algorithm proposed in this paper elaborately. Section 4 analyzes a headless rivet model, a roof truss structure and a composite cantilever beam structure to verify the effectiveness of the proposed method. Finally, conclusions are summarized in Section 5.

## 2. Review of the global reliability sensitivity analysis

Suppose the limit state function is

$$Y = g(\boldsymbol{X}) \tag{1}$$

where $\boldsymbol{X} = (X_1, X_2, \ldots, X_n)$ is the random input variable vector and $f_{\boldsymbol{X}}(\boldsymbol{x})$ is the joint PDF of model input variables. Given all the input variables are mutually independent, the joint PDF of model input variables can be expressed by a product of the marginal PDF $f_{X_i}(x_i)$ of $X_i (i = 1,2,\ldots,n)$, i.e., $f_{\boldsymbol{X}}(\boldsymbol{x}) = \prod_{i=1}^{n} f_{X_i}(x_i)$. The region $F = \{\boldsymbol{x}: g(\boldsymbol{x}) \leqslant 0\}$ is defined as the failure domain. Therefore, the failure probability of this structural system is expressed as follows:

$$P_f = \Pr\{g(\boldsymbol{X}) \leqslant 0\} = \int_{g(\boldsymbol{x}) \leqslant 0} f_{\boldsymbol{X}}(\boldsymbol{x}) d\boldsymbol{x} \tag{2}$$

To measure the effect of model input variables on the failure probability, Cui et al. and Li et al. [29,30] proposed the failure probability-based global sensitivity index also called global reliability sensitivity index which is depicted as follows:

$$\delta_i^P = E_{X_i}\left(P_f - P_{f|X_i}\right)^2 = \int_{-\infty}^{+\infty} \left(P_f - P_{f|X_i}\right)^2 f_{X_i}(x_i) dx_i \tag{3}$$

where $P_f$ is the unconditional failure probability and $P_{f|X_i}$ is the conditional failure probability when $X_i$ is fixed. $\delta_i^P$ reflects the average effect of the input variable $X_i$ on the failure probability of the model. The higher $\delta_i^P$ is, the more importance $X_i$ is on the failure probability.

Ref. [30] proved that Eq. (3) has the same form with the variance-based sensitivity index, i.e.,

$$\delta_i^P = E_{X_i}\left(P_f - P_{f|X_i}\right)^2 = V(E(I_F|X_i)) \tag{4}$$

where $I_F(\cdot)$ is the indicator function of the failure domain, i.e.,

$$I_F(\boldsymbol{x}) = \begin{cases} 1 & g(\boldsymbol{x}) \leqslant 0 \\ 0 & g(\boldsymbol{x}) > 0 \end{cases} \tag{5}$$

Wei et al. [31] standardized it by dividing Eq. (4) by the unconditional variance of the failure domain indicator function, i.e.,

$$S_i = \frac{V(E(I_F|X_i))}{V(I_F)} \tag{6}$$

Eq. (6) indicates that the failure probability-based global sensitivity index is the first order variance effect of input on the failure indicator function.

## 3. The proposed global reliability sensitivity analysis algorithm

Many existing global reliability sensitivity analysis methods not only need the unconditional model input-output samples but also require other conditional model input-output samples, which leads to the dimensional-dependency. In this section, an alternative sampling-based method is introduced to efficiently obtain the global reliability sensitivity indices of all the model input variables only by one group unconditional model input-output sample matrix.

### 3.1. The proposed equivalent expression of $S_i$ based on the Bayes formula

According to Bayes formula, the conditional failure probability $P_{f|X_i}$ can be equivalently expressed as

$$P_{f|X_i} = \frac{P_f f_{X_i}(x_i|F)}{f_{X_i}(x_i)} \tag{7}$$

where $f_{X_i}(x_i|F)$ is the failure-conditional PDF of $X_i$.

Thereof, Eq. (6) can be represented as