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# An innovative adaptive sparse response surface method for structural reliability analysis

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ABSTRACT

In the scope of infrastructure risk assessment, structural reliability analysis leads to a challenging problem in order to deal with conflicting objectives: accurate estimation of failure probabilities and computational efficiency. Since the application of classical reliability methods is limited and often leads to a prohibitive computational cost, metamodeling techniques (e.g. polynomial chaos, kriging, response surface methods (RSM), etc.) have been widely used. Nevertheless, existing RSM present limitations handling with highly non-linear limit states, large-scale problems and approximation error. To overcome these problems, this paper describes a cutting-edge response surface algorithm covering the following issues: (i) dimensionality reduction by a variable screening procedure; (ii) definition of a promising search domain; (iii) initial experimental design based on an optimized space-filling scheme; (iv) model selection according to a stepwise regression procedure; (v) model validation by a cross-validation approach; (vi) model fitting using a double weighted regression technique; (vii) sequential sampling scheme by exploring a defined region of interest; (viii) confidence interval of reliability estimates based on a bootstrapping technique. With the aim of proving its efficiency, a wide collection of six illustration examples, concerning both analytical and FE-based problems, was selected. By benchmarking obtained results with literature findings, proposed method not only outperforms existing RSM, but also provides a powerful alternative to the use of other metamodeling techniques.



## 1. Introduction

Due to the increasing need of fulfilling performance and safety requirements, which can be threatened by potentially hazardous events, risk and reliability assessment are a task of utmost importance in many complex engineering applications [1]. In this regard, both aleatory and epistemic uncertainty play an important role and their effects must be carefully treated. Aiming at studying the propagation of uncertainties, probabilistic approaches coupled with advanced numerical models are often required. In structural engineering's field, reliability assessment is a challenging task, particularly for complex structures with small failure probabilities [2–4]. On the one hand, to reproduce important structure behaviour's features, non-linear finite element analysis models may be required. On the other hand, computational costs increase substantially due to numerical model's complexity. In this context, a significant research focus has been placed at evaluating failure probabilities keeping computational effort at a reasonable level.

In structural reliability analysis, failure probability with respect to a limit state described by a performance function $G(\mathbf{x})$, representing the system response of interest, is computed by the following $n$-fold integral:

$$P_f = P\{G(\mathbf{x}) \leqslant 0\} = \int_{G(\mathbf{x})\leq 0} f_{\mathbf{x}}(\mathbf{x})\mathrm{d}\mathbf{x} \tag{1}$$

where $\mathbf{x}$ describes an $n$-vector of basic input random variables with its joint probability density function $f_{\mathbf{x}}(\mathbf{x})$ and $G(\mathbf{x}) \leq 0$ defines a failure event or state.

Unfortunately, failure probability evaluation in Eq. (1) is hindered by two major hassles: joint probability density function $f_{\mathbf{x}}(\mathbf{x})$ is unknown and the integration domain is defined implicitly. Hence, several probabilistic reliability methods have been developed in the past few decades [2–4]. Basically, these methods can be roughly classified into three different groups of increasing complexity: approximate or gradient-based methods, simulation or sampling-based methods and metamodeling-based methods. Herein, basic principles of the most important methods are briefly introduced.

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With respect to approximate methods, two of the most commonly used approaches are the so-called FORM and SORM (first and second order reliability methods). Basically, failure probability computation is facilitated by means of a three-step procedure: (i) simplification of the integrand $f_{\mathbf{x}}(\mathbf{x})$ through a transformation operation, in which random variables are mapped into a standard normal space $\mathbf{u}$; (ii) approximation of the integration boundary $G(\mathbf{u}) = 0$, based on a Taylor series expansion; (iii) determination of the "design point" through a search algorithm (or "most probable failure point" (MPFP), which is the point with the highest probability density on the limit state function (LSF) $G(\mathbf{u}) = 0$). With the aim of yielding a good computational performance, several optimization algorithms have been developed [5]. However, these methods present a few important drawbacks [2,3]: (i) gradients and Hessians may not be easily available; (ii) in the case of highly non-linear limit states at $\mathbf{u}$-space, linear approximation given by FORM can substantially compromise accuracy, which might be slightly improved by SORM; (iii) in failure probability estimation, approximation errors are not taking into account; (iv) in case of large-scale problems, finite difference scheme usually adopted to compute finite element response's gradients may affect severely the computational effort.

As for simulation-based methods, these comprise both crude and more advanced Monte Carlo (MC) schemes based on variance reduction techniques, such as, importance sampling (IS) and its variants [6,7], line sampling [8,9], directional sampling [10,11], Latin hypercube sampling (LHS) [12,13], subset simulation (SS) [14,15]. Although crude MC is the most robust approach to solve the integral in Eq. (1), its computational cost, when dealing with time-demanding performance evaluations and/or very reliable engineering systems, may be considered as prohibitive. Variance reduction techniques can efficiently improve MC estimators by exploring the vicinity of failure region. However, the reduction of performance evaluations still remains insufficient [16,1,17].

Aiming at overcoming these inefficiencies, metamodeling-based methods provide an interesting solution to enhance reliability analysis' efficiency by replacing the true performance function $G(\mathbf{x})$ with a metamodel (or surrogate model) $\widehat{G}(\mathbf{x})$, which is incomparably faster to be evaluated. In general, a metamodel is constructed from a collection of known input–output pairs, which constitute the so-called experimental design (ED). Various methodologies have been proposed by exploiting different kind of metamodels, such as polynomial response surfaces (RS) [18–25], polynomial chaos expansions (PCE) [26–28], artificial neural networks [29–32], support vector machines [33–35], kriging [36–38,17,39,40], among others. Regardless of the metamodel, these iterative methodologies consist of the following steps: (i) application purpose; (ii) metamodel goal; (iii) experimental design strategy; (iv) model fitting; (v) model validation; (vi) failure probability evaluation; (vii) model updating until convergence is reached. Indeed, both accuracy and efficiency strongly rely on how these issues are addressed. Aiming to facilitate comprehension and comparison between existing approaches, the most efficient methods for each kind of metamodel are roughly reviewed and compared in Table 1, according to their chronological order.

In agreement with this literature review, by taking advantage of its exact interpolation method and efficient active learning methodologies, kriging-based are the most efficient methods which have been largely improved during the last few years [37,38,17,39,40]. However, its practical implementation is not a straightforward task [36], since it requires special knowledge of statistics and optimum learning functions. In fact, this intensive research effort attempts to overcome the limitations of both classical and adaptive RSM. In spite of being less efficient than kriging-based methods, quadratic RS are still the most popular metamodels due to its compromise between practical applicability and efficiency. However, RS approaches suffer from the "curse of dimensionality", which is discussed in Section 2.1.

The main difference between kriging and RS as metamodels lies in their aim/goal, since two broad procedures can be followed [41]: regression or classification approach. On the one hand, regression technique uses a collection of statistical tools to mathematically model (e.g. quadratic RS, PCE) non-deterministic relationships between response of interest and explanatory variables. Depending on how these models are built, global interpolation accuracy may be compromised. On the other hand, even though classification procedure also investigates those relationships, its main focus is to identify and cluster different classes or categories among observations by means of statistics, machine learning or neural networks approaches. Mostly kriging-based methodologies understand reliability problem as a classification task, which is interested in classifying samples as either safe or unsafe depending on their location [41].

One of the keys to the success of a metamodeling-based method is the combination of metamodel's merits with an sequential experimental design strategy. This strategy must guide experiments to explore the design space of interest, which is examined in Sections 2.2 and 2.6. Generally, a sequential (or adaptive) procedure starts with an initial ED, which is successively enriched by additional training points until a stopping or convergence criteria is fulfilled. Numerous design of experiments techniques can be found in literature (see [42,16]), such as, star-shaped, factorial, central composite, space-filling designs (e.g. low-discrepancy sequences, LHS), optimal-design, among others. However, their sample size grows quickly with problem dimension, which directly affects the computational effort needed. This obstacle is addressed in Section 2.3.

Furthermore, with regard to RS methods, structural reliability estimates are strongly influenced by the choice of both their polynomial form and experimental design scheme [43]. To tackle this shortcoming, weighted regression approach [22], re-sampling technique to achieve design point's confidence area [27], sampling method based on partial derivatives of RS with respect to random variables [23], nested LHS designs [25] and moving-least squares approximations [44] have been developed. Herein, a double weighted technique to improve fitting accuracy near failure surface is presented in Section 2.5.

As for other metamodels, the majority of the approaches define an initial ED as equal to a Monte Carlo sample and additional subsequent promising training points are often selected according to learning functions [45,46] or by means of Monte Carlo Markov Chains (MCMC) algorithms [47], which depend on the previously selected initial design that might be updated.

Although the development of adaptive and refinement strategies coupling sampling-based methods with metamodeling has been widely studied, model validation and error propagation has not received much attention by the research community [17]. Indeed, evaluating metamodel predictability during validation stage is an essential step to ensure an adequate "degree of belief" about its use [48]. Despite some effort in this regard (see Table 1), the estimation of prediction error given by leave-one-out cross-validation (LOOCV), which is the simplest form of exhaustive cross-validation (CV), presents high variance due to the large overlap of training folds. Hence, aiming to achieve a better bias-variance trade-off: 10-fold CV or 10-times 10-fold CV may be preferable [49]. In this regard, a model validation procedure is highlighted in Section 2.4.

Beyond the estimation of metamodel's prediction error, its influence on reliability analysis estimates is also an issue of great importance. With the aim of assessing metamodel prediction