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State dependent Girsanov's controls in time variant reliability estimation in randomly excited dynamical systems

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ABSTRACT

The problem of time variant reliability estimation of structural dynamical systems subjected to nonstationary, Gaussian, random excitations is considered. The system equations are cast in the form of Ito's stochastic differential equations and the problem of reliability estimation is tackled based on Monte Carlo simulations with a Girsanov transformation based sampling variance reduction scheme. The problem of time variant reliability analysis is first cast as an equivalent problem in series system reliability analysis. Novel contribution of the work lies in proposing procedures to arrive at state dependent (closed loop) Girsanov's controls. Suboptimal Girsanov's controls for estimating the time variant reliability are derived based on component level ideal controls, which are exactly obtainable for linear systems, and, via a local linearization step for nonlinear systems. It is shown that a simplified version of the above closed loop controls, that avoids linearization step for nonlinear systems, can be deduced by minimizing a distance measure similar to what has been done for arriving at open loop controls. Illustrations on multi-degree of freedom linear/nonlinear systems demonstrate the superior performance of the proposed method *vis-à-vis* the existing open loop control based methods. Limited largescale Monte Carlo simulations are used to verify the acceptability of solutions based on the proposed scheme.

1. Introduction

Problems of time variant reliability estimation in randomly driven structural systems are widely encountered in earthquake, wind, and automotive engineering. Here, one considers the response vector $\boldsymbol{X}(t)$ of the system and aims to determine the reliability $P_S = P[h[\boldsymbol{X}(t)] < h^*, \forall t \in [0, T]]$, which is the probability that a scalar function $h[\boldsymbol{X}(t)]$ of the response vector $\boldsymbol{X}(t)$ stays below a specified threshold h^* for all times in the interval $[0, T]$. Here $P[\cdot]$ denotes the probability measure. The complement, $P_F = 1 - P_S$, denotes the probability of failure. An exact solution to this problem is rarely possible and the complexity of the problem increases as one deals with strong structural nonlinearity, large state-space dimensions, nonstationary and (or) non-Gaussian excitations, parametric excitations, randomness in system parameters, and nonlinear nature of the performance measure $h[\boldsymbol{X}(t)]$. Approximate analytical/combined analytical-numerical approaches include those based on counting the number of times a specified level is crossed by a random process [1–3], those which adopt Markov vector models for response processes and tackle the problem of reliability through the use of the backward Kolmogorov equations [4–7], and methods based on probability density evolution method [PDEM] and extreme value theory [8–13]. The following points may be noted in this context:

(a) The methods based on counting level crossings employ Poisson/Markovian models for the level crossings and the underlying approximations are valid only in an asymptotic sense. When applied to nonlinear systems, this approach typically requires the application of equivalent linearization or closure approximations [1,14].
(b) The methods based on the application of backward Kolmogorov equation are feasible to be applied only for low dimensional systems.
(c) The PDEM is more generally applicable, but, is not suited to treat excitations which are modeled as white noise or filtered white noise processes. Such representations for loads are often employed in engineering problems [15]. This approach does not take advantage of the extensive knowledge base that is available in the existing literature on the theory and applications of Markov processes.

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The Monte Carlo simulation based methods, on the other hand, are eminently suited to handle the above mentioned complexities. These methods, however, need to be reinforced with suitable strategies to control the sampling variance, in the absence of which, the methods prove to be infeasible, especially, when dealing with estimation of probabilities associated with rare events, such as, structural failures. The simulation methods can be broadly grouped into two categories: the first which begin by discretizing the random excitations into a set of equivalent random variables and subsequently perform the reliability calculations in the space spanned by these discretized set of random variables [16–21] and, the second, which work with time trajectories of the response/excitation processes [22–25]. In the first approach, the time variant problem gets converted to an equivalent static/time-invariant reliability problem which is subsequently tackled using importance sampling methods or Markov chain Monte Carlo techniques. Here, the difficulty arises when the dimension of the random vector increases due to the large number of random variables entering the formulation, and one has to devise special sampling strategies to tackle this issue. The second approach, on the other hand, is bereft of such complexities and requires one to develop techniques to artificially manipulate the response trajectories $h[\boldsymbol{X}(t)]$ during simulation to drive them towards the failure region.

Among the second class of methods, those based on applying the Girsanov's transformation of probability measure are mathematically well-founded [26–28]. Here, the governing equation of the dynamical system is cast as an Ito's stochastic differential equation (SDE) and the response trajectories are nudged towards the failure region through the addition of artificial control forces in the dynamical system. The process leads to a change in the underlying probability measure which is accounted for by introducing a correction term in the failure probability estimator. Although there exists an ideal control resulting in a zero sampling variance failure probability estimator [27], deducing it becomes infeasible as it requires knowledge of P_F, the very quantity we are trying to estimate. Hence, existing studies have focused on development of suboptimal controls. Tanaka [22] and Macke and Bucher [25] have obtained suboptimal controls for the time variant reliability problem by minimizing a distance function, as in the first order reliability methods [29]. The study by Schoenmakers et al. [30], proposes an application of the Kolmogorov backward equation to obtain the controls. Olsen and Naess [31] have developed an iterative procedure, based on design point oscillations and optimal stochastic control theory, to obtain suboptimal Girsanov's controls for single-degree of freedom (sdof) oscillators. Strategies to design the control forces using ideas from critical excitations [32,33] and through stochastic optimization [34], by solving the Bellman equation, have been studied by Au for the case of sdof elasto-plastic oscillators. It may also be noted that studies reported in references [31–34] consider only sdof oscillators under stationary Gaussian excitations. In recent years, methods for Girsanov's transformation based time variant reliability analysis of randomly parametered nonlinear systems [35], time variant system reliability analysis via vibration testing [36], and reliability model updating [37] have also been developed.

Most of the above mentioned studies have employed open loop (state independent) Girsanov's controls which, by their very nature, are pre-computed and deterministic. These controls cannot adapt their actions according to the random behavior of the dynamical system under consideration. It is important to note that the ideal (zero sampling variance) control, stated above, is a closed loop (state dependent) control and is stochastic in nature. Thus, it appears that the open loop controls, because of their deterministic nature, have limited potential to approach the ideal controls no matter with what detail they are designed. With this in mind, the present study explores strategies to devise closed loop controls for estimating time variant reliability with reduced sampling variance. Specifically, we propose two alternatives to tackle the problem: one that builds on the form of the ideal controls, and, the other, which is a simplified version valid for random excitation intensity asymptotically approaching zero. Illustrations include studies on multi-degree of freedom (mdof) systems under nonstationary, multi-component random excitations. The trustworthiness of the results obtained, based on the proposed procedure, has been examined using pertinent results from direct Monte Carlo simulation.

2. Objectives of the present study

We consider the class of dynamical systems which are governed by Ito's SDE of the form

$$\mathrm{d}\boldsymbol{X}(t) = \boldsymbol{A}[\boldsymbol{X}(t), t]\mathrm{d}t + \boldsymbol{\sigma}[\boldsymbol{X}(t), t]\mathrm{d}\boldsymbol{B}(t);\ \boldsymbol{X}(0) = \boldsymbol{X}_0;\ 0 \leqslant t \leqslant T \tag{1}$$

Here $\boldsymbol{X}(t)$ is a $p \times 1$ vector of system states, $A[\boldsymbol{X}(t), t]$ is the $p \times 1$ drift vector, $\boldsymbol{\sigma}[\boldsymbol{X}(t), t]$ is the $p \times q$ matrix of drift coefficients, and $\boldsymbol{B}(t)$ is a $q \times 1$ vector of Brownian motion processes with $\mathrm{E_P}[\Delta\boldsymbol{B}(t)] = \mathrm{E_P}[\boldsymbol{B}(t+\Delta t) - \boldsymbol{B}(t)] = 0$ and $\mathrm{E_P}[\Delta B_i(t_1)\Delta B_j(t_2)] = C_{ij}\Delta t\delta(t_1 - t_2)$ for $0 \leqslant t_1, t_2 \leqslant T$ and $i, j = 1, \ldots, q$. We write $C_{ij} = \rho_{ij}\sqrt{S_i S_j}$ where $S_i, i = 1, \ldots, q$, denote the intensities of the underlying Gaussian white noise processes and the correlation coefficients ρ_{ij} satisfy $|\rho_{ij}| \leqslant 1$. The initial condition in Eq. (1) is assumed to be deterministic. We denote by $(\Omega, \mathscr{F}, \mathrm{P})$ the underlying probability space, and by $\mathrm{E_P}[\cdot]$ the expectation operator with respect to the probability measure P. In the context of finite element models for structural systems, this equation is taken to be obtained by recasting the semi-discretized equations resulting from discretization in space. The class of systems represented by this model is quite general: it can accommodate nonlinear systems, nonstationary, non-white and (or) non-Gaussian excitations, and parametric and (or) external excitations. In case of non-white excitations, the forcing functions are obtained as outputs of additional filters which are driven by white noise excitations. In such cases, the system state vector $\boldsymbol{X}(t)$ includes additional state variables associated with the augmented filter equations. Furthermore, by making these filters nonlinear in nature, one can allow for excitations to be non-Gaussian.

A scalar measure of system performance, denoted by $h[\boldsymbol{X}(t)]$, and a corresponding permissible threshold h^*, are now introduced. This function, for example, could be in terms of allowable limits on displacements, reactions transferred, or stress metrics. The problem of time variant reliability consists of evaluating

$$\begin{aligned} P_F &= 1 - \mathrm{P}[h[\boldsymbol{X}(t)] < h^* \forall t \in [0, T]] \\ &= \mathrm{P}\left[\left\{h^* - \max_{0<t\leqslant T} h[\boldsymbol{X}(t)] \leqslant 0\right\}\right] \end{aligned} \tag{2}$$

which is the probability of occurrence of the failure event $\mathrm{F} = \left\{h^* - \max_{0<t\leqslant T} h[\boldsymbol{X}(t)] \leqslant 0\right\}$. According to the method of Girsanov's transformation [28], in order to obtain a variance reduced estimator for P_F, we modify Eq. (1) by introducing an additional control force leading to the modified equation

$$\begin{aligned} \mathrm{d}\tilde{\boldsymbol{X}}(t) &= A\left[\tilde{\boldsymbol{X}}(t), t\right]\mathrm{d}t + \boldsymbol{\sigma}\left[\tilde{\boldsymbol{X}}(t), t\right]\boldsymbol{u}\left[\tilde{\boldsymbol{X}}(t), t\right]\mathrm{d}t \\ &\quad + \boldsymbol{\sigma}\left[\tilde{\boldsymbol{X}}(t), t\right]\mathrm{d}\tilde{\boldsymbol{B}}(t);\ \tilde{\boldsymbol{X}}(0) = \boldsymbol{X}_0, 0 \leqslant t \leqslant T \end{aligned} \tag{3}$$

where, $\boldsymbol{u}\left[\tilde{\boldsymbol{X}}(t), t\right]$ is the $q \times 1$ vector of control force and $\tilde{\boldsymbol{B}}(t)$ is an Ito's process given by

$$\mathrm{d}\tilde{\boldsymbol{B}}(t) = -\boldsymbol{u}\left[\tilde{\boldsymbol{X}}(t), t\right]\mathrm{d}t + \mathrm{d}\boldsymbol{B}(t);\ \tilde{\boldsymbol{B}}(0) = 0;\ 0 \leqslant t \leqslant T \tag{4}$$