



Hyper-spherical extrapolation method (HEM) for general high dimensional reliability problems

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ABSTRACT

To tackle challenges in low-probability, high-dimensional reliability analysis, this paper proposes hyper-spherical extrapolation method to estimate the failure probabilities efficiently and accurately. The extrapolation method employs hyper-spherical formulations of reliability problems developed based on geometric insights of high dimensional standard normal space. The proposed method can extrapolate the low probability region of interest using failure probabilities obtained from high/median probability region. Owing to the generality of the formulation, the proposed method is expected to work for general, component and system reliability problems defined in high-dimensional space of random variables. Using different presumptions on extrapolation, two slightly different versions of the extrapolation method are developed. Numerical examples with analytical limit-state functions and those concerning nonlinear random vibration of a hysteretic oscillator are investigated to test and demonstrate the performance of the proposed method. Finally, to facilitate an in-depth understanding of the proposed method and further developments, insights gained from the development of the method are also provided. The supporting source code and data are available for download at <https://github.com/ziqidwang/Hyper-spherical-extrapolation-method>.

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1. Introduction

As the scope of risk and reliability analysis expands, various reliability methods are being applied to increasingly larger-scale problems involving hundreds or thousands of random variables. It is well known (see [1,2]) that some of the classical reliability methods (e.g. first/second-order reliability methods (FORM/SORM), design-point-based importance sampling methods) face difficulties in such high dimensional reliability problems and could produce erroneous results. The reason has to do with the exponentially increasing volume of the probability space with the number of random variables, and the fact that in high dimensions most of the contribution to the failure probability comes from far regions in the failure domain that have small probability density but large volume [2].

Compared with other reliability methods, the performance of the brute-force Monte Carlo sampling (MCS) scheme employing the original joint probability density function has the merits of being irrelevant to dimension and complexity of the limit-state functions. However, since most reliability problems are

characterized by small failure probabilities, the brute-force MCS is computationally inefficient and thus practically infeasible to solve reliability problems with low failure probability. More advanced Monte Carlo simulation techniques (e.g. sequential Monte Carlo method [3–5], subset simulation [6–8], and cross-entropy based sampling method [9–11]), although being more efficient than the brute-force MCS, are still characterized by relatively large computational demands for high dimensional, complex reliability problems.

In this paper, the issues in low-probability, high dimensional reliability problems are addressed by developing an extrapolation method that uses failure probabilities obtained from high/median probability levels. The extrapolation method is developed using geometric insights of high dimensional standard normal space in conjunction with a framework of hyper-spherical formulations for reliability problems. This “hyper-spherical extrapolation” method is designed to work for general, component and system reliability problems defined in high dimensional spaces.

The structure of this paper is as follows. In Section 2, theoretical concepts that are useful to understand the proposed extrapolation method are first introduced. Section 3 provides general concepts and computational details of the proposed method. In Section 4, numerical examples with analytical limit-state functions as well

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as those representing nonlinear random vibration of a hysteretic oscillator are investigated to test and demonstrate the method. Section 5 provides discussions and insights on the development of the extrapolation method. Finally, concluding remarks are presented in Section 6.

2. Formulation of reliability problems in high dimensional standard normal space

The failure probability of a reliability problem defined in the n -dimensional uncorrelated standard normal space can be formulated in terms of the distance from the origin [11], i.e.

$$P_f = \int_0^\infty \theta(r) f_\chi(r) dr \quad (1)$$

in which $f_\chi(r)$ is the probability density function (PDF) of the χ -distribution with n degrees of freedom representing the random distance from the origin, and $\theta(r)$ is the failure ratio of the hypersphere surface area with radius r . The failure ratio $\theta(r)$ represents the percentage of the surface area of the hypersphere with radius r that belongs to the failure domain. In a standard normal space, points on a hypersphere is uniformly distributed, thus integrating the failure ratio $\theta(r)$ times the PDF $f_\chi(r)$ over the distance r yields the failure probability.

Following Eq. (1) and using Monte-Carlo sampling, the probability in Eq. (1) can be estimated by

$$P_f \cong \frac{1}{M} \sum_{i=1}^M \theta(r_i) \quad (2)$$

where r_i 's, $i = 1, \dots, M$, are random samples drawn from the χ -distribution $f_\chi(r)$ with n degrees of freedom.

Note that Eqs. (1) and (2) are valid for any dimensions, however, they are especially useful in high dimensional problems. This is because in a high dimensional standard normal space almost all 'probability information' (consider the term $\theta(r)f_\chi(r)dr$ in Eq. (1)) is concentrated in a relatively narrow "important ring" region with radius $\sqrt{n} \pm \varepsilon$, where n is the dimension and ε is a perturbation which is small compared to $\sqrt{n}$ [2]. A hypersphere that closely encompasses the mode (i.e. the point in the probability space at which its probability density has a locally maximum value) has high probability density, but in high dimensions the surface area of the hypersphere is negligible compared with a hypersphere that is far away from the mode. Therefore, in high dimensional Gaussian space the contribution to the probability involves a trade-off between the exponentially decrease in probability densities with the distance from the mode and the exponentially increase in the spherical area with the distance from the mode. As a consequence of this trade-off, there exists a typical set (important ring) where the densities integrated by the volume makes dominant contributions to the probability. For example, in a standard normal space with a dimension of 400, more than 95% of the probability is contained within the hyper ring of width 20 ± 1 and more than 99.99% of the probability is contained within the hyper ring of width 20 ± 2 . In fact, it is shown that when $n \rightarrow +\infty$, $r \sim N(\sqrt{n}, 1/2)$ [2]. Thus a random distance r_i (drawn from $f_\chi(r)$) in Eq. (2) is highly likely to have $r_i \in [\sqrt{n} - \varepsilon, \sqrt{n} + \varepsilon]$. As a result, if the dimension is relatively high in Eq. (2) the variation in $\theta(r_i)$ would be small.

This paper employs the concept of the *hyper-spherical failure domain*. In an n -dimensional standard normal space, the *hyper-spherical failure domain* with radius r is defined as the intersection between the failure domain and the hypersphere with radius r , i.e.

$$\mathcal{F}_r \equiv \{S_r \cap \mathcal{F}\} \quad (3)$$

in which $\mathcal{F}_r$ denotes the hyper-spherical failure domain with radius r , S_r denotes a hypersphere with radius r , and $\mathcal{F}$ is the failure domain of the reliability problem. Note that $\mathcal{F}_r$, S_r and $\mathcal{F}$ are all defined in the n -dimensional standard normal space. For a reliability problem with linear limit-state surface in the n -dimensional standard normal space, $\mathcal{F}_r$ is circular (given S_r intersects $\mathcal{F}$). For more general reliability problems, $\mathcal{F}_r$ can be arbitrary shape and can have multiple modes.

3. Development of hyper-spherical extrapolation method (HEM)

3.1. General concepts of the hyper-spherical extrapolation method

Consider a *hyper-spherical cap* on a hypersphere with radius r defined in an n -dimensional Euclidian space, which is mathematically expressed by

$$S_{cap}(r, \boldsymbol{\mu}, \alpha) = \{\mathbf{u} \in \mathbb{R}^n | \mathbf{u}^T \boldsymbol{\mu} \geq r^2 \cos \alpha, \|\mathbf{u}\| = \|\boldsymbol{\mu}\| = r\} \quad (4)$$

where $\boldsymbol{\mu}$ is the direction vector representing the center of the cap, α is the maximum angle between the center $\boldsymbol{\mu}$ and vectors on the hyper-spherical cap. The term $\cos \alpha$ can be alternatively written as $\cos \alpha = b/r$, where b is the distance between the hypersphere center and center of the base of the hyper-spherical cap (see Fig. 1).

The surface area of the hyper-spherical cap $S_{cap}(r, \boldsymbol{\mu}, \alpha)$ can be expressed by [12]

$$A_{cap}(r, \alpha) = \frac{1}{2} A_{sp}(r) B_{\sin^2 \alpha} \left(\frac{n-1}{2}, \frac{1}{2} \right), \quad \alpha \in [0, \frac{\pi}{2}] \quad (5)$$

where $B_{\sin^2 \alpha}(\frac{n-1}{2}, \frac{1}{2})$ is a regularized incomplete beta factor, and $A_{sp}(r)$ is the surface area of the hypersphere with radius r . Eq. (5) is derived by integrating the surface area of an $(n-1)$ -hypersphere with radius $r \sin \alpha$ over a great circle arc, and using properties of the beta function. Details of the derivation can be found in [2]. Note that Eq. (5) is valid for $\alpha \in [0, \pi/2]$, yet the extension to other cases is straightforward. The regularized incomplete beta factor $B_{\sin^2 \alpha}(\frac{n-1}{2}, \frac{1}{2})$ can be interpreted as the probability of a random vector uniformly drawn from a hemisphere (with $\boldsymbol{\mu}$ pointing the center of the hemisphere) falling onto the cap. For example, $\alpha = \pi/2$ corresponds to a probability of 1, and $\alpha = 0$ corresponds to a probability of 0.

If the hyper-spherical cap is used to represent a hyper-spherical failure domain, using Eq. (5) the failure ratio $\theta_{cap}(r, \alpha)$ can be written as

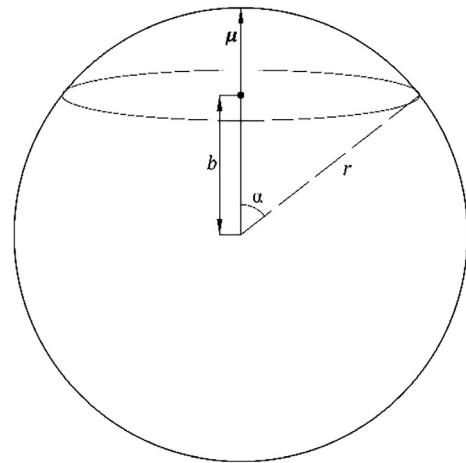


Fig. 1. Hyper-spherical cap.

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