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An optimized strategy for using asymptotic sampling for reliability analysis

Christian Gasser *, Christian Bucher

Center of Mechanics and Structural Dynamics, Technische Universität Wien, Karlsplatz 13, Vienna, Austria

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ABSTRACT

Asymptotic sampling is a recently developed method for calculating small failure probabilities of high-dimensional problems. It is based on Monte Carlo simulation, however, extremely small probabilities can be estimated with reasonable computational effort. The present paper presents an optimal strategy for using asymptotic sampling. In particular, it is explained in which cases the accuracy of the results can be increased by applying low-discrepancy sampling. A new surrogate technique is presented for obtaining accurate results also in cases in which low-discrepancy sampling is not applicable effectually. Furthermore, an extension of the method is shown for estimating threshold exceedance probabilities.

1. Introduction

The safety of structures depends on resistances and loads, which both eventually have to be regarded as random by nature. All parameters which influence the resistance of the structure (cross-section values and material properties) as well as all kinds of loads acting on the structure can be considered as random variables described by their respective probability density functions. In this way, one obtains a multidimensional joint probability density function:

$$f_{X_1,\ldots,X_n}(x_1,\ldots,x_n) \tag{1}$$

in which X_i are the random variables and n is the dimension. Failure is typically denoted in terms of a scalar limit state function $g(.)$ attaining negative values. The failure probability P_F can then be expressed as an integral, bounded by the limit state function:

$$P_F = \int_{g(x_1,\ldots,x_n)\le 0} \cdots \int f_{X_1,\ldots,X_n}(x_1,\ldots,x_n)dx_1,\ldots,dx_n \tag{2}$$

The challenge in calculating this integral lies in evaluating the limit state function, which for nonlinear systems usually requires an incremental/iterative numerical approach [1]. Since usually just a very small region contributes to the value of the integral, it is difficult to place integration points for numerical integration procedures appropriately [1]. This is particularly true for high-dimensional integrals.

One method to overcome this difficulty is Monte Carlo (MC) simulation. Thereby, the integral is evaluated by repeated random sampling. The failure probability is then expressed as

$$P_F = \frac{N_{fail}}{N_{eval}} \tag{3}$$

Here N_{eval} is the number of evaluations of the limit state function and N_{fail} is the number of evaluations which resulted in a failure.

The generalized reliability index β is defined by

$$\beta = \phi^{-1}(1 - P_F) \tag{4}$$

In this equation, $\phi^{-1}(.)$ is the inverse standardized Gaussian distribution function. In the following, it will be assumed that the random variables X_i are independent and identically distributed (i.i.d.) Gaussian variables with zero mean and standard deviation σ. Non-Gaussian variables and possible correlations among them may be introduced using marginal transformations and joint probability density function models such as the Nataf-model [2,3,1].

The major advantage of MC for integral calculation lies in its independence of the dimension. This means that the efficiency of the method is not impaired by the number of variables the limit state function depends on. However, since a representative number of failures is needed, the number of function evaluations becomes prohibitively large when small failure probabilities are

* Corresponding author.
E-mail address: christian.gasser88@gmail.com (C. Gasser).

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to be calculated. For example, to calculate a failure probability of the order of 10^{-5}, the number of evaluations should be larger than 10^6. Hence, one can easily imagine, that MC becomes impracticable when a complicated finite element model is to be analysed.

Different strategies have been developed to reduce the number of necessary function/model evaluations. Importance sampling methods [1] aim at obtaining more failures by shifting the sampling to the small region in n-dimensional space which actually contributes significantly to the failure probability. An alternative approach is to express the failure probability as a product of larger conditional probabilities by introducing intermediate failure levels. Thereby, the samples are drifted closer to the failure space in each level. This method is known as subset simulation [4]. Recently, another method, called asymptotic sampling, has been presented in [5].

2. General concept of asymptotic sampling

Similarly to the previously mentioned approaches, asymptotic sampling (AS) aims at obtaining a representative number of failures out of a moderate number of function evaluations. However, the method relies on a certain asymptotic property of the failure probability in the n-dimensional i.i.d. normal space.

Asymptotic sampling exploits the asymptotic behaviour of the failure probability expressed in terms of the reliability index β when changing the standard deviation of the basic random variables. If the original standard Gaussian random variables are replaced by variables with non-unit standard deviations $\sigma = \frac{1}{f}$, then the computed reliability index will depend on the choice of f. As a first simple case, consider a linear function of the basic random variables X_k, say

$$g(\mathbf{X}) = \sum_{k=1}^{N} a_k X_k \tag{5}$$

with arbitrary real-valued coefficients a_k. The random variable $Y = g(\mathbf{X})$ will then be Gaussian with a zero mean and a variance

$$\sigma_Y^2 = \sum_{k=1}^{N} a_k^2 \tag{6}$$

The distribution function $F_Y(\xi)$ of this variable will therefore be given by

$$F_Y(\xi) = \Phi\left(\frac{\xi}{\sigma_Y}\right) = \Phi\left(\xi \Big/ \sqrt{\sum_{k=1}^{N} a_k^2}\right) \tag{7}$$

Upon changing the standard deviation of all basic variables from unity to a value of $1/f$, the standard deviation of Y changes by the same amount. Thus, the distribution function changes to

$$F_Y(\xi) = \Phi\left(\xi f \Big/ \sqrt{\sum_{k=1}^{N} a_k^2}\right) \tag{8}$$

from which the generalized reliability index (with respect to scale factor f) is immediately found as

$$\beta(f) = \frac{\xi f}{\sqrt{\sum_{k=1}^{N} a_k^2}} \quad \rightarrow \quad \frac{\beta(f)}{f} = \frac{\xi}{\sqrt{\sum_{k=1}^{N} a_k^2}} = \text{const.} \qquad (\text{for fixed } \xi) \tag{9}$$

This means that for a linear function of Gaussian variables, the expression $\beta(f)/f$ is invariant with respect to the choice of f. Geometrically, β is the minimum distance of the origin from the limit surface $g(\mathbf{X}) = 0$.

Eq. (9) is not true for the general, i.e. nonlinear, case. There is, however, an asymptotic property which ensures similar behaviour for many nonlinear cases. Eq. (15b) of [6] states that the generalized reliability index asymptotically converges to the minimum distance of the origin from the limit surface as $\beta \rightarrow \infty$. Hence the above relation holds asymptotically as f approaches infinity:

$$\lim_{f \rightarrow \infty} \frac{\beta(f)}{f} = \text{const.} \tag{10}$$

This asymptotic property is exploited to construct a regression model which allows to determine the reliability index for extremely small failure probabilities. Fig. 1 shows this concept. By increasing the standard deviation of the basic variables, a representative number of failure events can be obtained out of a moderate number of simulation runs. The scaled reliability indices obtained in that way are used as support points for the regression. The reliability index of the original system, i.e. $\beta(f = 1)$, is then obtained by extrapolation.

For the fitting process the functional dependence is chosen as

$$\frac{\beta(f)}{f} = A + \frac{B}{f^C} \tag{11}$$

Assuming C to be positive, this approach ensures asymptotic convergence to a constant value (here to A) as $f \rightarrow \infty$ (which is equivalent to $\sigma \rightarrow 0$). Thus, the asymptotic property given in Eq. (10) is satisfied. Of course, any other approach maintaining the asymptotic behaviour could be chosen as well. The coefficients A, B and C can be determined from a regression analysis, typically by a least-squares fitting [5].

3. Optimizing the algorithm

In [5] it has been shown that asymptotic sampling is independent of the dimensionality. However, accuracy of the approach depends on several other factors, namely:

- accuracy of support points
- number of support points
- collocation of support points
- regression parameters
- particular geometrical shape of the limit state surface.

Obviously, the geometrical shape of the limit state surface of a specific problem in standard normal space cannot be influenced by the analyst. Thus, a substantial part of this paper focuses on strategies to increase the accuracy of the support points. Furthermore, advices concerning number and collocation of the support

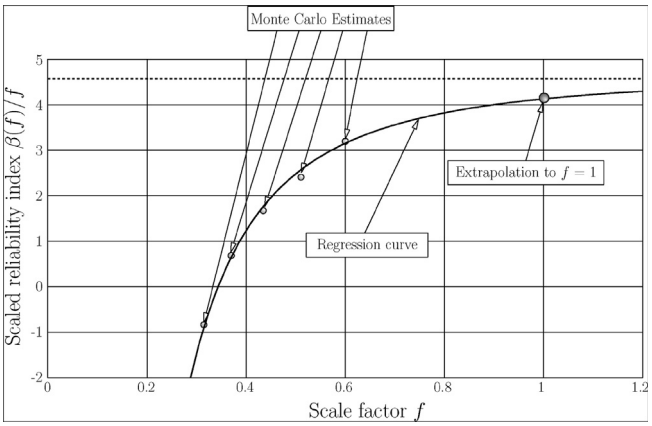

Fig. 1. Basic concept of asymptotic sampling.