State-dependent stochastic models: A general stochastic framework for modeling deteriorating engineering systems considering multiple deterioration processes and their interactions

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ABSTRACT

For performance analysis of deteriorating engineering systems, it is critical to model and incorporate the various deterioration processes and associated uncertainties. This paper proposes state-dependent stochastic models (SDSMs) for modeling the impact of deterioration on the performance of engineering systems. Within the stochastic framework, the change of the system state variables due to different deterioration processes and their interaction is modeled explicitly. As a candidate model to be used in the framework, a new general age and state dependent stochastic model for gradual deterioration is proposed, and its calibration based on data is also discussed. Once the time-variant system state variables are modeled, proper capacity and demand models that take them as inputs can be adopted to fully capture the impact of deterioration processes on the capacity, demand, and other system performances. The proposed framework is first demonstrated through a simple example, and then is used to model the deterioration of an example reinforced concrete (RC) bridge considering deterioration caused by both corrosion and earthquake including their interaction. The results show the importance of modeling the interaction between different deterioration processes, and also verify the advantages of the proposed framework.

1. Introduction

Engineering systems suffer from deterioration over time due to either aging, regular operation, or extreme loading/environmental conditions [1–4]. In general, two types of deterioration mechanisms can be identified in most engineering systems: gradual (progressive) deterioration (e.g., due to corrosion, alkali silica reaction, fatigue, crack growth) [3,5–8] and shock (sudden) deterioration (e.g., due to damages from earthquakes, hurricanes, floods, blasts and other natural or anthropogenic hazards) [1,9]. Deterioration may considerably reduce the service life and reliability of these systems. Therefore, it is critical to model and incorporate deterioration in the analysis, design, and operation of engineering systems, especially in reliability analysis [10–16], life-cycle analysis [17–21], remaining service life/lifetime prediction [13,22], maintenance/operation [23–25] when the time-variant properties and performances of the systems play an important role.

In [1], a stochastic framework for modeling deterioration is proposed, where the deterioration process is modeled as a combination of shock and gradual process. The framework accounts for the effects of deterioration on both capacity of the system and demands imposed on the system by external loads, and also facilitates consideration of two general types of failure, namely failure due to a cumulative damage (or a deterioration process exceeding a permissible limit) [9] and failure due to an engineering demand parameter exceeding the corresponding capacity [10,24,26]. The stochastic framework in [1] is based on three assumptions to facilitate the semi-analytical solution of the probability distributions for the number of shocks to failure and failure time. The framework assumes that (i) the shock and gradual deterioration processes are independent of each other; (ii) the shock deterioration is independent of the state variables (e.g., material, geometrical and structural properties) of the system at the time of the shock and the sequence of shock deteriorations are modeled using statistically independent and identically distributed (*s.i.i.d*) random variables; and (iii) the gradual deterioration is represented with a deterministic function.

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This paper generalizes the stochastic framework in [1] by removing the need for the above assumptions. To allow for this generalization, a new general formulation is proposed for modeling the change of the system state variables (such as material properties, member dimensions, imposed boundary conditions, etc.) due to different deterioration processes. Specifically, by focusing on the change of the state variables and including contributions from different deterioration processes, this formulation explicitly considers the interaction between different deterioration processes [addressing assumption (i)]; the change of the state variables due to shock deterioration is modeled as state dependent (i.e., the change is dependent on the state variables at the time of the shock) [addressing assumption (ii)]; the change of the state variables due to gradual deterioration is modeled as age/time and state dependent [addressing assumption (iii).] The proposed formulation is general and can be used with different deterioration models (such as deterministic, probabilistic, and stochastic models). Overall, a more general stochastic framework for modeling impact of deterioration is established by integrating the new formulation for modeling change of state variables. Once the time-variant state variables of the system are modeled, they can be used in existing capacity and demand models that take them as inputs to fully propagate the effects of deterioration to both capacity of the system and demands imposed on the system by external loads. As a candidate model that can be used in the proposed formulation, a general age and state dependent stochastic model for gradual deterioration extended based on the model in [27] is proposed. To facilitate its calibration based on observation data, a stochastic sampling based approach is proposed to establish the likelihood function (within the context of Bayesian updating). Two examples are presented. First, a simple example is presented to illustrate the proposed formulation for modeling the change of the state variables and the importance of capturing the joint impact of multiple deterioration processes. Then, the proposed general framework is used to estimate the time-variant fragility and failure probability of an example reinforced concrete (RC) bridge considering deterioration caused by both chloride-induced corrosion and earthquakes including their interaction (e.g., impact of earthquakes on corrosion initiation and corrosion rate are explicitly modeled). The results show the advantages of the proposed framework, and provide valuable insight on the impact of corrosion and earthquakes on the performance of RC bridges.

This paper is organized as follows. Section 2 presents a review of current approaches for deterioration modeling. Section 3 presents the proposed general framework and the proposed mathematical formulation for modeling the changes in the state variables due to deterioration processes, including both shock and gradual deteriorations. Section 4 discusses stochastic models for the changes in the state variables that can be used in the mathematical formulation described in Section 3 as well as their calibration (including both gradual and shock deterioration). The estimation of time-variant system performances is discussed in Section 5. Section 6 presents two examples to illustrate the proposed framework. The last section summarizes the research findings.

2. Review of deterioration modeling

During its lifetime, a system may be subject to multiple deterioration mechanisms. Incorporation of these deteriorations in the evaluation of the system performance has been studied by many researchers. Existing research focuses primarily on a specific deterioration mechanism, i.e. either gradual [3,28] or shock deterioration [29,30] separately. Fewer and more recent studies have considered both gradual and shock deterioration [1,9,18] where

the two deterioration types are considered to be independent. However, in general different deteriorations interact with each other [16,31–33].

As to the specific models for deterioration, considering the stochastic nature of deterioration and to account for the various uncertainties, stochastic process based models have been gaining popularity [23,34,35] (for example Markov process models, including continuous-time Markov processes such as Wiener process, Gamma process, inverse Gaussian process and discrete-time ones having finite or countable deterioration state space). However, most existing research has focused on deterioration with a single (scalar) performance measure; also these models assume independent deterioration increments (whether it is for gradual or shock) and are usually purely age/time dependent, while in reality the increments may also largely depend on the system state. Though some age and state dependent model has been proposed [27], it only considers scalar state variable and the state dependence is only on this scalar variable, while more general models need to consider vector of state variables with deterioration of each state variable potentially dependent on the values of all the state variables. In addition to the state dependence, deteriorations are also impacted by the environmental conditions (such as temperature, pressure, humidity, chlorides availability) and loading conditions which usually change over time as well (nonstationary in nature) [11,36]. Systems (and even different components of the same system) in different environmental conditions may have different deterioration rates. Further research is needed in developing general stochastic models that can truthfully characterize the actual deterioration processes. The establishment of such models and their calibration based on experimental or monitoring data are important topics that need to be tackled.

Furthermore, existing research has focused primarily either on modeling deterioration directly at the performance level, e.g., predicting the reduction of structural capacity/resistance [1,9–11,30,37] or on understanding experimentally and modeling the physics of deterioration processes, e.g., predicting the changes in mechanical or structural properties [3,38,39]. Although for engineering practice we are ultimately interested in the system performance, to establish models at the performance level, issues related to their calibration and how to estimate the performance at the first place have not been fully addressed and need further research. There is a need to establish and calibrate deterioration models at the mechanical and structural properties level and then propagate the changes in the mechanical and structural properties to compute the resulting changes at performance level.

Overall, there still lacks a general and versatile framework to model deterioration that incorporates the impacts of multiple deterioration processes and their interactions, and integrates all available information including data (typically available at the component or material level) and physics-based models of the structural properties. The framework proposed here aims to address the aforementioned limitations.

3. Proposed general framework for modeling impact of deterioration

3.1. Overall framework

Fig. 1 illustrates the proposed general framework for the modeling of the impact of deteriorations considering multiple deterioration processes and their interactions. The framework starts with the vector of the external conditions/variables at time t, $\mathbf{Z}(t)$. Furthermore, $\mathbf{Z}(t)$ is partitioned into (i) environmental conditions/variables such as temperature, atmospheric pressure, relative humidity, etc., denoted as $\mathbf{E}(t)$, and (ii) shock measures such as