Structural Safety 70 (2018) 93–103

Contents lists available at ScienceDirect

Structural Safety

journal homepage: www.elsevier.com/locate/strusafe

Extreme value distribution and small failure probabilities estimation of structures subjected to non-stationary stochastic seismic excitations

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ARTICLE INFO

Article history:
Received 18 September 2016
Received in revised form 1 November 2016
Accepted 17 October 2017

Keywords:
Extreme value distribution
Small failure probabilities
Kernel density estimation
Least-square fitting
Parametric model
Non-stationary stochastic seismic excitations

ABSTRACT

A new method is proposed for efficient estimating the extreme value distribution (EVD) and small failure probabilities of structures subjected to non-stationary stochastic seismic excitations. This method first involves a preliminary estimation by kernel density estimation (KDE), which oscillates across the true probability density function (PDF), as the original data for fitting. The selection of bandwidth in KDE is suggested. Then, two least-square fitting procedures are performed to reconstruct the EVD, where a two-section form parametric model for the EVD is proposed. The shifted generalized lognormal distribution (SGLD), which has a rich flexibility in shape, is fitted based on the preliminary estimation to obtain the main body of EVD. On the other hand, the tail distribution of EVD can be obtained by fitting the probability of exceedance (POE) curve in logarithmic coordinate via a quadratic equation. Two numerical examples, involving both linear and highly nonlinear structures subjected to non-stationary stochastic seismic excitations are investigated. The EVDs and POE curves obtained by direct KDE and the proposed method are all compared with those by Monte Carlo simulation (MCS). The investigations indicate the accuracy and efficiency of the proposed method.

1. Introduction

The extreme value distribution (EVD) of response of structures subjected to seismic excitations, which can be modeled as non-stationary stochastic processes, is of paramount importance in earthquake engineering, particularly in seismic reliability assessment and risk analysis [1,2]. In fact, the first-passage probability of response is equivalent to the corresponding exceedance probability of extreme value distribution of response at a specified threshold [3,4]. In other words, the first-passage reliability of structures subjected to seismic excitations can be estimated once the EVD of response is determined.

For the estimation of EVD of response, analytical methods can be used in very special cases and hence are not applicable to general engineering problems. Alternatively, some approximate methods have been developed in the past several decades for this problem. One of the well-known results is the level-crossing process based method, which takes the Rayleigh distribution as the EVD of a narrow-banded Gaussian stochastic process [5]. However, the Poisson's assumption or the Vanmarcke's assumption of the level-crossing event is usually imposed for stochastic processes, which may come from intuition and empirical data rather than theoretical basis [1]. Some other approximate methods include the equivalent linearization method [6], Fokker-Planck equation method [7], moment closure method [3], and so on. These methods have been well developed, however, there are still great difficulties to apply these methods for practical engineering structures exhibiting strong nonlinearity under seismic loadings. The third is the simulation based method, which is suitable to general nonlinear structures driven by stochastic seismic excitations. Although Monte Carlo simulation (MCS) is generally usable to derive the EVD, the practical applicability might reach soon the limits of the feasibility due to large computational efforts in the case of complex numerical models [8]. The moment method, which requires neither iterations nor the computation of derivatives, is effectively extended to derive the EVD of structural dynamic systems [9], where the moments are evaluated efficiently based on the dimension reduction methods. However, this method becomes quite unwieldy if the size of the random vector increases and when

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https://doi.org/10.1016/j.strusafe.2017.10.007
0167-4730/

seismic excitations are modeled as non-stationary stochastic processes. In the past nearly 20 years, a new method named probability density evolution method (PDEM) [10], which can capture the instantaneous probability density function (PDF) of general nonlinear multiple-degree-of-freedom (MDOF) structures with randomness involved in both structural properties and excitations, is developed and extended to estimate the EVD of response [1,11]. In PDEM, the physical modelings of stochastic dynamic excitations [12,13] are usually employed, where some physical parameters are modeled as random variables. This method serves as a powerful tool to estimate the PDF and EVD of dynamic response of structures with high accuracy and efficiency [14–21]. Nevertheless, the efficient methods for estimating the EVD of nonlinear structures are still highly desirable.

On the other hand, the efficient estimate of small failure probabilities of nonlinear structures under random excitations is always a challenging task [22]. In this regard, the tail distribution of EVD should be modeled with accuracy. Several techniques for modeling the tail distribution of EVD, e.g. the subset simulation method [23], tail equivalent linearization method [24] and asymptotic sampling method [25], etc., have been developed recently. However, these techniques may not be able to keep the good trade-off of accuracy and efficiency for the tail distribution modeling. He and Gong propose an extrapolation method to efficiently obtain the tail of EVD [2,22], where two relatively large exceedance probabilities need to be specified based on direct simulations for parameters estimation. Nonetheless, the selection of different exceedance probabilities may significantly influence the accuracy of tail distribution modeling.

In the present paper, we will develop an efficient method for estimating the EVD and small failure probabilities of general nonlinear structures subjected to non-stationary stochastic seismic excitations. The paper is arranged as follows: In Section 2, the problem formulation is elucidated. Then, Section 3 devotes to developing a two-step method to reconstruct the main body and tail distribution of EVD, respectively, in which the kernel density estimation and two least square fitting procedures are involved. The efficacy of the proposed method is illustrated by two numerical examples in Section 4, including a linear and a hysteretic MDOF structures subjected to non-stationary stochastic seismic excitations. In Section 5, the concluding remarks are included.

2. Problem formulation

Consider an n-degree-of-freedom general nonlinear structures subjected to non-stationary stochastic seismic excitations. The equation of motion is governed by

$$\mathbf{M}\ddot{\mathbf{X}}(t) + \mathbf{C}\dot{\mathbf{X}}(t) + \mathbf{G}[\mathbf{X}(t)] = -\mathbf{MI}\ddot{x}_g(t) \tag{1}$$

where $\mathbf{M}$ and $\mathbf{C}$ are the n by n mass and damping matrices and $\mathbf{X}$, $\dot{\mathbf{X}}$ and $\ddot{\mathbf{X}}$ are the n by 1 displacement, velocity and acceleration vectors, respectively. The overdots stand for derivative with respect to time. $\mathbf{G}[\mathbf{X}(t)]$ is the restoring force vector, which might be linear or nonlinear with respect to $\mathbf{X}$. The term $\ddot{x}_g(t)$ is the time-domain representation of acceleration of ground motion, which is regarded as a non-stationary stochastic process.

In the past decades, the modeling of non-stationary stochastic process has attracted great attention, which is of great significance for seismic reliability analysis of general nonlinear structures. A variety of methods have been well studied for this purpose, among which the spectral representation method [26] is widely adopted in practical engineering. The time-domain representation of non-stationary stochastic process $\ddot{x}_g(t)$ by the spectral representation method is given by

$$\ddot{x}_g(t) = g(t)\left[\sqrt{2}\sum_{i=1}^{m} A_i\cos(\omega_i t + \theta_i)\right] \tag{2}$$

where $g(t)$ is the envelope function defined as

$$g(t) = \begin{cases} (t/t_1)^2, & 0 \leqslant t < t_1 \\ 1, & t_1 < t \leqslant t_2 \\ \exp[-c(t-t_2)], & t_2 < t \leqslant T \\ 0, & t \geqslant T \end{cases} \tag{3}$$

where c is the coefficient of attenuation, t_1 and t_2 are the start and end time instants of the stationary portion of the ground motion, T is the time duration of the ground motion.

The amplitude A_i is represented by

$$A_i = \sqrt{2S_{\ddot{x}_g}(\omega_i)\Delta\omega} \tag{4}$$

where $S_{\ddot{x}_g}(\omega)$ is the earthquake acceleration power spectral density (PSD), $\Delta\omega$ is the frequency interval and $\omega_i = i\Delta\omega$, ω_is are the discretized frequencies.

The random phase angles θ_is are the independent random variables uniformly distributed over $[0, 2\pi]$.

Actually, the infinite series is involved in the spectral representation method and thus the truncation must be performed practically. To make the error of using the spectral representation method to describe the stochastic seismic excitation as small as possible, many terms, say m = 500–1000, are usually retained. It is seen that the truncation of the infinite series also results in a large number of random variables involved in Eq. (1) to obtain the stochastic seismic response of structures.

If we denote $\boldsymbol{\Theta} = (\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \ldots \boldsymbol{\theta}_m)$, the solution to Eq. (1) can be represented in the form that

$$\mathbf{X}(t) = \mathbf{H}(\boldsymbol{\Theta}, t), \dot{\mathbf{X}}(t) = \mathbf{h}(\boldsymbol{\Theta}, t) \tag{5}$$

where $\mathbf{H}$ and $\mathbf{h}$ are deterministic operators.

Usually, a one-dimensional physical quantity such as the stress at key point, the inter-storey drift, etc., is of great concern for reliability analysis, which could be expressed as

$$Z(t) = \psi[\mathbf{X}(t), \dot{\mathbf{X}}(t)] = H_Z(\boldsymbol{\Theta}, t) \tag{6}$$

where ψ and H_Z are other deterministic operators.

Denote the extreme value of response $Z(t)$ for the structural dynamic system (1) as

$$Z_{ext} = \underset{t\in[0,T]}{ext} Z(t) = W(\boldsymbol{\Theta}, T) \tag{7}$$

Usually, the extreme value Z_{ext} is a positive random variable. For example, if one considers the maximum absolute value of $Z(t)$ in the time interval $[0, T]$, Eq. (7) is equivalent to

$$Z_{ext} = |Z|_{\max} = \max_{t\in[0,T]}|Z(t)| \tag{8}$$

The reliability R in the form of extreme value can be expressed as [1]

$$R = \Pr\{Z_{ext} \leqslant Z_B\} \tag{9}$$

where Pr denotes probability for short, Z_B is the threshold.

Equivalently, Eq. (9) can be transformed as

$$R = \int_0^{Z_B} p_{Z_{ext}}(z)dz \tag{10}$$

where $p_{Z_{ext}}(z)$ is the extreme value distribution (EVD).

The failure probability p_f is then goes to

$$p_f = 1 - R \tag{11}$$

It is seen that the EVD of response plays a crucial role in assessing the reliabilities or the failure probabilities of structures. In this