Structural Safety 70 (2018) 104–114

Contents lists available at ScienceDirect

Structural Safety

journal homepage: www.elsevier.com/locate/strusafe

Non-Gaussian parameter estimation using generalized polynomial chaos expansion with extended Kalman filtering

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ARTICLE INFO

Article history:
Received 9 May 2016
Received in revised form 11 October 2017
Accepted 24 October 2017

Keywords:
Polynomial chaos expansion
Parameter estimation
Uncertainty quantification
Kalman filer
Extended Kalman filter

ABSTRACT

Kalman Filter (KF) based parameter estimation assumes Gaussianity of the system parameters and thus propagates only the first two moments of the states. Application of Particle filter or Ensemble Kalman filter to estimate non-Gaussian parameters, although more accurate, is computationally expensive. Generalized polynomial chaos (gPC) is well-known as an effective tool to describe any dynamic system with stationary uncertainty through a set of orthogonal basis functions and associated coefficients. This article couples gPC with Extended KF (EKF) algorithm in which the uncertainty propagation from parameter to measurement is described through gPC expansion of parameters and outputs. Subsequently, the gPC coefficients of the parameter expansion are estimated from available measurements employing EKF. Thus, instead of selecting the system parameters as states, we consider the associated parameter gPC coefficients as state variables which reduces the problem of estimating the complete distribution of parameters down to identification of a few gPC coefficients. The proposed method is tested on systems with either Gaussian or non-Gaussian parameters. The error in estimating non-Gaussian parameters using KF based techniques is demonstrated.

1. Introduction

Response prediction of a complex structural system is generally achieved through an idealized mathematical model based initially on a set of prior assumptions and then updated periodically with new information obtained through measurements. The initial idealization and subsequent updating impart uncertainty in the model and its predictions. To enhance the predictive ability of the model, systematic calibration through inverse estimation of parameters from the real measurements is often practised. Commonly, uncertainties in the model parameters are dealt with in a probabilistic framework where variability in the measurement space is mapped back to the parameter space. These types of problems can be categorised under the broad class of stochastic inverse problems.

Direct identification of parameter uncertainty from output variability information requires the simulator model to be invertible, which is not always assured. To reduce computational complexity and time, approximating the actual simulator model by a meta model can be used. Unfortunately, replacing a detailed phenomenological model with a much simplified meta model increases the model uncertainty.

1.1. Existing methods

Identification of parametric uncertainty in probabilistic framework can be performed using Bayesian inference through maximum likelihood estimation (MLE) [1–4]. In MLE approach, the Bayesian estimation problem is posed as a large dimensional optimization problem and subsequently solved using gradient based [5] or other optimization techniques [6,7]. However, due to the non convex nature of this high dimensional problem, obtaining a practical solution often poses as the major challenge.

Kalman filtering [8] (KF) based stochastic data assimilation techniques have been applied extensively to identify system parameter uncertainty from noisy output measurements [9–13] by considering the parameters as additional Gaussian states. KF attempts Bayesian belief propagation to optimally estimate the system states by combining prior belief on states with its likelihood with new measurement. Being a linear estimator, application of KF is limited only to linear systems. This shortcoming led to the introduction of the nonlinear variants of KF (e.g. Extended KF (EKF) [14,15], Unscented KF (UKF) [16] etc.) to handle nonlinear problems by either locally linearising the system or imposing Gaussianity on the posterior distribution. EKF performs first order Taylor series expansion of the state transition functions, while UKF propagates the uncertainty through a set of weighted sigma points

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https://doi.org/10.1016/j.strusafe.2017.10.009
0167-4730/

around the current state estimate. To employ EKF/UKF for parameter estimation, parameters are appended in an extended state vector while representing the otherwise linear system through a bi-linear/nonlinear state space representation [17]. Nevertheless, the assumption on Gaussianity in states or parameters might not always agree with the real situation. Forcibly fitting a Gaussian distribution to a non-Gaussian parameter, in fact, can produce large errors in its estimation.

To accommodate non-Gaussian distributions, particle filter (PF) approaches propagate higher order moments through a set of particles [18–21] and subsequently the posterior estimate is obtained by updating the prior estimate of the particles with their respective likelihood with the current measurement. PF assumes that the parameter domain is discrete and thus updating the prior probability of a discrete set of sample particles using their likelihood gives a measure of the uncertainty in the predefined parameter set. However, with increased dimensionality in the parameter space, the computational demand increases heavily which can render PF computationally inefficient [22]. Apart from these filtering techniques, crude Monte-Carlo sampling based Ensemble Kalman filtering technique offers a robust approach to identifying the parametric variability [23]. However, estimating the probability distribution for the entire set of parameters accurately can be quite expensive.

1.2. Generalized polynomial chaos expansion (gPC)

Introduced by Spanos and Ghanem [24] using the concepts given by Wiener [25] as homogeneous chaos expansion, Polynomial chaos expansion (PCE) technique has emerged as an efficient tool to describe systems with stationary uncertainty using a set of orthogonal bases and associated coefficients [26,27]. PCE can be considered as an advancement of Karhunen–Loeve(KL) expansion [28,29] to discretize any random quantity and to describe its uncertainty through parametrization since the former does not demand the covariance function of the random space to be known a priori. Xiu and Karniadakis [30] later generalized PCE (denoted as gPC) using the result of Cameron-Martin [31] to discretize arbitrary random spaces using hypergeometric orthogonal polynomials chosen from the so called Askey scheme.

In gPC, the physical random variable χ is expressed in terms of a random vector ξ, termed as germ. Based on the selection of germ distribution, a set of mutually orthogonal basis functions (polynomials) $\phi(\xi)$ can be selected. With this germ ξ and polynomial bases $\phi(\xi)$, the physical random variable χ is described as:

$$\chi(\xi) = a_0\phi_0(\xi_0) + \sum_{i_1=1}^{\infty} a_{i_1}\phi_{i_1}(\xi_{i_1}) + \sum_{i_1=1}^{\infty}\sum_{i_2=1}^{\infty} a_{i_1,i_2}\phi_{i_1,i_2}(\xi_{i_1},\xi_{i_2}) + \cdots\infty = \sum_{i=1}^{\infty} a_i\phi_i(\xi) \tag{1}$$

where a_is are the coefficients of the polynomial expansion.

Verlaan and Heemink [32] used the intrusive Galerkin projection approach to solve the coefficients a_i of the expansion. In later works, collocation technique [33] introduced a more efficient approach for estimating the polynomial coefficients.

The basis polynomials vary depending on the germ distribution. For example, for a normally distributed germ, Hermite polynomials are the best suited basis functions, for uniformly distributed germ the basis should be Legendre type and so on. Details of other polynomial basis for different germ distributions are listed in Table 1. To describe a random variable exactly through gPC, an infinite order expansion is ideally required. However, for the sake of practicality, the expansion is generally truncated beyond a certain order.

gPC has been employed by Xiu and Karniadakis [34–36] for solving stochastic differential equations of fluid mechanics problems. Sandu et al. [37–39] employed the gPC technique for multibody dynamics and parameter estimation problems [40]. Soize and Ghanem [41] demonstrated it for estimating arbitrary probability densities. Desceliers et al. [42] employed maximum likelihood estimate (MLE) to identify the gPC coefficients of an arbitrary random filed.

Table 1
Polynomial types for different germ distributions.

Germ distribution	Support domain	Polynomial type
Normal $N(0,1)$	$\mathbb{R}$	Hermite
Uniform $u(-1,1)$	$[-1,1]$	Legendre
Gamma	$\mathbb{R}^+$	Laguerre
Beta	$\mathbb{R}^+$	Jacobi

Pence et al. [54], Pence [55] employed a combination of gPC and MLE in which each point estimate on the probable solution grid is propagated though the system dynamic model using gPC and subsequently MLE is employed to identify the estimate. Jacquelin et al. [56] proposed a modification in gPC to accelerate its convergence. gPC theory has also been extensively used in the literature for uncertainty propagation of otherwise deterministic systems [25,30,43]. This technique is capable of describing an arbitrary parameter distribution in an inexpensive way. It has been extensively used in the context of structural mechanics problems as well [44–48,59–61]. A review of its application for structural vibration problems can be found in Schuëller and Pradlwarter [49]. Sepahvand et al. [50,51] employed gPC for the purpose of parametric uncertainty quantification of stochastic systems. Blanchard et al. [40,52] used gPC technique along with EKF algorithm for parameter identification: gPC solves the forward dynamic state-space problem while EKF updates the state estimates. Li and Xiu [53] demonstrated the application of Ensemble Kalman filtering (EnKF) with gPC theory: the computational efficiency and accuracy are increased by solving the state prediction equation through gPC.

In this article, we couple gPC with Extended Kalman Filter (EKF), to propose a new algorithm in which the uncertainty propagation from parameter to measurement is described using a gPC meta model. The required gPC coefficients of the parameter gPC model are then estimated inversely using EKF. As we show in the following, such an approach enables an accurate and efficient estimation of any random parameter.

2. A new parameter estimation approach

2.1. The problem formulation

Let the system be characterized by a set of parameters $\mathbf{x}$ that are random in nature. There is a map $\mathbb{F}$ (possibly unknown but accurate models of which are available) that relates $\mathbf{x}$ to the system output $\mathbf{y}$. The actual output $\mathbf{y}$ is not known, but can be measured as $\bar{\mathbf{y}}$ repeatedly, giving the collection $\bar{\mathbf{Y}}$. The objective of this study is to determine the probability distribution of $\mathbf{x}$ using the information stored in $\bar{\mathbf{Y}}$ and the best available model $\mathbb{F}$. Algorithm 1 displays the pseudo-code of the coupled gPC-EKF estimator developed in this work.

$\mathbb{F}$ is typically a finite element model that maps each realization in parameter space $\mathbf{x}$ to a corresponding point in output space $\mathbf{y}$. Thus for any parameter-output pair, if the uncertain parameter can be described by a gPC expansion with a set of germs as its argument, the associated output gPC expansion can always be defined by the same set of germs:

$$\mathbf{y}(\underline{\xi}) = \mathbb{F}(\mathbf{x}(\underline{\xi})) \tag{2}$$

where $\mathbf{x}(\underline{\xi}) = [x_1(\xi_1), x_2(\xi_2), \ldots, x_n(\xi_n)]$ is the parameter vector and $\underline{\xi} = [\xi_1, \xi_2, \ldots, \xi_n]$ is the associated germ vector.