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System reliability with correlated components: Accuracy of the Equivalent Planes method

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ABSTRACT

Computing system reliability when system components are correlated presents a challenge because it usually requires solving multi-fold integrals numerically, which is generally infeasible due to the computational cost. In Dutch flood defense reliability modeling, an efficient method for computing the failure probability of a system of correlated components – referred to here as the Equivalent Planes method – was developed and has been applied in national flood risk analysis. The accuracy of the method has never been thoroughly tested, and the method is absent in the literature; this paper addresses both of these shortcomings. The method is described in detail, including an in-depth discussion about the source of error. A suite of system configurations were defined to test the error in the Equivalent Planes method, with a focus on extreme cases to capture the upper bound of the error. The 'exact' system reliability was computed analytically for the special case of equi-correlated components, and otherwise using Monte-Carlo directional sampling. We found that errors in the system failure probability estimates were low for a wide range of system configurations, and became more substantial for large systems with highly-correlated components. In the most extreme cases we tested, the error remained within three times the true failure probability. We provided an example of how one can determine if such error is tolerable in their particular application. We also show the computational advantage of using the Equivalent Planes method; large systems with small failure probabilities which take over 17 h for Monte Carlo directional sampling were computed with the Equivalent Planes in less than one second.

1. Introduction

System reliability analysis investigates the probability that a system will maintain its functionality; that is, the probability that the system will not fail. Computing the failure probability of complex systems, where the components within the system are correlated, usually requires multi-fold integrals, which are generally impossible to evaluate analytically. Consider a vector of random variables, $\mathbf{X} = [x_1, x_2, \ldots, x_n]$, containing both load and strength variables. The failure of the system is represented by the n-fold integral:

$$P_f = \int_{\Omega(\mathbf{X})} f_{\mathbf{X}}(X)dX, \tag{1}$$

where $f_{\mathbf{X}}(X)$ is the multivariate density function of X, and $\Omega(\mathbf{X})$ is the failure space, consisting of all realizations of X that lead to failure of the system. The configuration of the failure space depends on how the components in the system are connected: in series, in parallel, or in some hybrid combination. When connected in series, which is typical in levee systems, $\Omega(\mathbf{X}) = \bigcup_i Z_i(\mathbf{X}) < 0$, where $Z_i(\mathbf{X})$ is the limit state function of the ith component, and where failure of each component is defined by $Z_i(\mathbf{X}) < 0$. Monte Carlo methods to estimate the integral in (1) are typically prohibitively slow, especially in cases where evaluating the limit state functions requires calls to finite element models.

A number of methods have emerged in the past decade to address the need for efficient methods to compute system reliability. Sues and Cesare ([1]) proposed a method (Most Probable Point System Simulation, or MPPSS) in which the reliability of the system components is first computed via a method that returns a closed form of the limit state function (e.g. first- or second-order reliability methods). The limit state functions, together with the Boolean expressions defining failure, are then sampled in a Monte Carlo framework. The authors claim that the size of the system is trivial

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because of the closed form of the limit state functions, but for highly reliable components and/or large systems, it can require billions of samples to acquire the desired accuracy, making this method potentially prohibitively time-consuming. Naess et al. ([2]) proposed a Monte-Carlo-based method in which some tail properties of the distributions are used to substantially improve efficiency. In a follow-up paper ([3]), they tested the method on a large system with thousands of components and found an uncertainty band in which the upper bound is approximately five times the failure probability of the lower bound, for 200,000 samples and a computation time of about 30 min to an hour. The method has not yet been tested on systems in which the limit state function requires calls to an intensive external model (e.g. a finite element model), but will most likely be prohibitively slow given the number of samples required. Kang and Song ([4]) proposed an efficient method (sequential compounding method, or SCM) in which the reliability of the components is first computed, and the components are subsequently combined into equivalent components, two at a time, until the full system reliability is obtained. They tested their method on various system configurations, and found very good accuracy for all the configurations considered in the paper. Chun et al. ([5]) presented a complimentary method to SCM, which computes the sensitivity of the system failure probability to the reliability indexes of the components. The method does not consider the sensitivity of the system failure probability to the random variables that influence the component reliability indexes.

In the Netherlands, the reliability of flood defense systems has been a key research area for decades. Based on a series of papers from the 1980s ([6–9]), an efficient method for combining the failure probabilities of correlated components – referred to here as the Equivalent Planes method – was developed for series systems and implemented in reliability software for the Dutch flood defense system ([10,11]). We want to emphasize that the method was designed for series systems (as flood defense systems are primarily connected in series); two components connected in parallel within a system that is primarily connected in series poses no problem, but the method is not intended for systems of numerous components all connected in parallel. Similar to the MPPSS method of Sues and Casare ([1]), the Equivalent Planes method first computes the failure probability of the components, and then replaces their limit state functions with closed-form expressions for subsequent combining. While the MPPSS method allows generic mathematical formulation, the Equivalent Planes method is restricted to linearized forms of the limit state function (hyperplanes). In contrast to the MPPSS method, the Equivalent Planes method does not rely on Monte Carlo methods. Similar to the Sequential Compounding method from Kang and Song ([4]), the Equivalent Planes method combines components sequentially; they differ most notably in the method to derive the correlation between a combined component and the remaining system components. To accomplish this, the Equivalent Planes method requires information about the autocorrelation of the underlying random variables contributing to failure; the Sequential Compounding method only requires the correlation between components.

The Equivalent Planes method was developed to simultaneously meet two requirements for Dutch flood defense reliability modeling: fast computation for large highly-reliable systems, and the ability to compute influence coefficients of both the random variables and the components. These influence coefficients are critical in Dutch flood defense reliability modeling on two fronts: (1) in deltas, where the flood defense system is subjected to loads fluctuating at different time scales, the influence coefficients are needed to scale the failure probability from the time scale of the highest-fluctuating load to the time scale of interest ([11]), and (2) they give flood defense managers a clear overview which variables, levee segments, or failure mechanisms are contributing the most to the failure probability and require the most attention.

In the Netherlands, the results of the method – the failure probability of a system of flood defenses – have been used in national flood risk analysis, on which major decisions about the safety standards of the defenses have been based ([12–14]). However, the accuracy of the Equivalent Planes method for large systems has never been well investigated. Additionally, although the method is in long-standing use, it remains absent from the literature. This paper serves thus two purposes. The first is to document the method in the literature, and the second is to set up a suite of academic system configurations which we can use to investigate the accuracy of the method.

The paper is laid out as follows. We first describe the Equivalent Planes method in Section 2; we then discuss the source of error in the Equivalent Planes method in Section 3; in Section 4 we describe the various system configurations that we define for investigating error propagation and show the performance of the Equivalent Planes method for these systems; we discuss the idea of tolerable error in Section 5, and close with discussion and conclusions in Section 6.

2. Equivalent Planes method

The Equivalent Planes method computes the failure probability (P_f) of a system of two correlated components, and – by applying it iteratively – the failure probability of a system of any number of components. The ith component is described by a limit state function, Z_i; failure occurs whenever $Z_i < 0$. The method starts with two components, connected in parallel (Eq. (2)) or in series (Eq. (3)). Often these components are correlated; that is, failure of one component will influence the failure probability of the second component.

$$P_f = P(Z_1 < 0 \cap Z_2 < 0) = P(Z_1 < 0) \cdot P(Z_2 < 0 | Z_1 < 0) \tag{2}$$

$$P_f = P(Z_1 < 0 \cup Z_2 < 0) = P(Z_1 < 0) + P(Z_2 < 0) - P(Z_1 < 0 \cap Z_2 < 0) \tag{3}$$

The strategy of the Equivalent Planes method is to replace the conditional probability $P(Z_2 < 0 | Z_1 < 0)$ with an equivalent marginal distribution $P(Z_2' < 0)$ which incorporates the condition $Z_1 < 0$ by having a non-zero density only in the failure space of component 1.

We will describe how the equivalent marginal distribution is computed. But first we will highlight the required information for getting started.

2.1. Getting started

To apply the Equivalent Planes method, we need to know the failure probability of each of the individual components and the correlation between component failures. The latter is driven by common variables. For example, consider a levee section along a river with two failure modes – overtopping and internal erosion; the water level in the river will influence the failure probability of both components, creating correlation between them. To compute the correlation between components, we need information about the variables that cause the correlation: (i) their *autocorrelation* – the correlation between a variable in component 1 and the same variable in component 2 – and (ii) *influence coefficients*, which describe how strongly each variable contributes to failure.

The autocorrelation of the variables can be equal to one in some cases (e.g. variables – like water level – which contribute to different failure modes at the same location will be the same for each failure mode). In other cases (consider soil permeability in two