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Multi-variate seismic demand modelling using copulas: Application to non-ductile reinforced concrete frame in Victoria, Canada

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ABSTRACT

Joint probabilistic characteristics of key structural demand variables due to intense ground shaking are important for quantitative seismic loss estimation. Current damage–loss models require inputs of multiple seismic demand parameters, such as maximum/residual inter-storey drift ratio (ISDR) and peak floor acceleration (PFA). This study extends current seismic demand estimation methods based on incremental dynamic analysis (IDA) by characterising dependence among different engineering demand parameters (EDP) using copulas explicitly. The developed method is applied to a 4-storey non-ductile reinforced concrete (RC) frame in Victoria, British Columbia, Canada. The developed multi-variate seismic demand model is integrated with a storey-based damage–loss model to assess the economic consequences due to different earthquake loss generation modes (i.e. non-collapse repairs, collapse, and demolition). Results obtained from this study indicate that the effects of multi-variate seismic demand modelling on the expected seismic loss ratios are significant. The critical information is the limit state threshold for demolition. In addition, consideration of a realistic dependence structure of maximum and residual inter-storey drift ratios can be important for seismic loss estimation as well as for multi-criteria seismic performance evaluation.

1. Introduction

An accurate assessment of potential impact of future destructive earthquakes is essential for effective disaster risk reduction. Probabilistic seismic risk analysis (PSRA) entails the state-of-the-art understanding of regional seismic hazard information, such as possible scenarios and likelihood of destructive shaking intensity, and seismic vulnerability of structures, such as damage accumulation and loss generation [1–4]. Using probabilistic calculus, PSRA evaluates the potential damage and loss that a certain group of structures is likely to experience due to various seismic events. Two key components in PSRA are structural capacity modelling and seismic demand characterisation. A structural model that is used in the assessment is required to be capable of simulating a wide range of structural behaviour from damage initiation to collapse. In particular, realistic representation of ultimate damage states and failure modes is of critical importance. The complexity and hysteretic characteristics of structural systems in interaction with ground motions having different amplitudes and frequency content result in large uncertainty associated with seismic fragility. Several studies have attempted to quantify such uncertainty and assessed their impact on structural response prediction [5–7].

Parameterisation of earthquake damage and loss generation processes has major influence on the computation and modelling of EDP that is adopted as structural response variable for damage and loss assessment. Typical EDP parameters include the maximum ISDR and PFA for structural and non-structural components [8,9]. In addition to transient EDP parameters, residual ISDR may be a critical parameter in determining the usability of damaged structures in a post-earthquake situation [10–12]. In PSRA, EDP is either uni-variate or multi-variate. When a scalar parameter that correlates well with damage severity is employed, detailed probabilistic models are developed using seismic demand estimation methods, such as IDA [13]. The multi-variate case is often implemented using fragility models for different types of damage sensitivity (e.g. drift-sensitive versus acceleration-sensitive). However, fragility curves for different EDP parameters are evaluated separately and thus dependence of EDP variables for a given seismic intensity measure (IM; e.g. spectral acceleration) is not taken into account explicitly. Ruiz-Garcia and Miranda [14] and Ramirez and Miranda [9] highlighted that inclusion of residual drift as EDP, in addition to maximum ISDR and PFA, can have major impact on the economic consequence due to earthquake damage, because a

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severely-damaged building may be demolished due to expensive repair costs. Moreover, performance matrices based on both maximum and residual ISDRs (denoted by MaxISDR and ResISDR, respectively) have been proposed for use in seismic damage assessment [11]. It is noteworthy that in the above-mentioned studies, dependence of MaxISDR and ResISDR, which are physically inter-related and thus statistically correlated, has not been elaborated. Goda [15] and Uma et al. [16] investigated the joint probabilistic modelling of the two inter-related parameters using inelastic single-degree-of-freedom systems. However, rigorous evaluation of joint probability distributions of MaxISDR and ResISDR for realistic multi-degree-of-freedom systems has not been carried out. Therefore, further investigations of joint probabilistic modelling of multiple EDP parameters are warranted to consider different modes of damage and loss generation.

This study investigates the joint probabilistic modelling of multiple EDPs by conducting detailed characterisations of marginal probability distributions for MaxISDR, ResISDR, and PFA and copula models between MaxISDR and ResISDR. A copula technique offers a flexible way of describing nonlinear dependence among multi-variate data in isolation from their marginal probability distributions, and serves as a powerful tool for modelling nonlinearly-interrelated multi-variate data [17,18]. The method is applied to a 4-storey non-ductile RC building located in Victoria, British Columbia, Canada. Seismic vulnerability of pre-1970 buildings constructed in British Columbia remains to be a major concern because of the use of older design codes and poor construction practices (e.g. lack of column confinement and poor detailing) at the time of design and construction [19,20]. Moreover, Victoria is situated in an active seismic region, affected by complex regional seismicity due to shallow crustal earthquakes, deep inslab earthquakes, and mega-thrust Cascadia subduction earthquakes [21,22]. Seismic demand modelling is conducted based on IDA by developing a probabilistic relationship between IM and EDP. To avoid bias due to excessive record scaling in assessing seismic performance of a structure, a multiple conditional mean spectra (CMS) method is implemented by reflecting regional seismic hazard characteristics in British Columbia [23,24]. The developed multi-variate seismic demand model is then integrated with a storey-based damage–loss model for non-ductile RC frames [9] to evaluate the effects of incorporating ResISDR in PSRA and dependence modelling between MaxISDR and ResISDR on earthquake loss generation (including demolition). The novel contributions of this study are: (i) copula-based multi-variate modelling of EDP parameters is developed for a realistic structural model, and (ii) the impact of multi-variate seismic demand modelling is assessed in terms of expected seismic loss and seismic performance metrics. The former essentially extends the current IDA-based seismic demand modelling approaches.

The paper is organised as follows. A brief summary of copula modelling is presented in Section 2. Section 3 introduces an overall seismic risk analysis framework (Section 3.1), followed by descriptions of finite-element modelling of the 4-storey non-ductile RC frame (Section 3.2), regional seismic hazard information in British Columbia (Section 3.3), IDA and seismic demand modelling (Section 3.4), and storey-based damage–loss assessment (Section 3.5). In Section 4, results of multi-variate seismic demand modelling for the non-ductile RC frame in Victoria are discussed, and its effects on seismic loss are evaluated quantitatively. Finally, main conclusions from the investigations are mentioned in Section 5.

2. Dependence modelling using copulas

Consider the joint probability distribution of two random variables X_1 and X_2, $H(x_1,x_2) = P[X_1 \leqslant x_1, X_2 \leqslant x_2]$, continuous marginal probability distributions of which are denoted by $F_1(x_1)$ (=u_1) and $F_2(x_2)$ (=u_2), respectively. u_1 and u_2 represent a sample of a standard uniform random variable U_1 and U_2, respectively, and $P[\bullet]$ represents the probability. Sklar's theorem dictates that a relationship among $H(x_1,x_2)$, $F_1(x_1)$, and $F_2(x_2)$ can be established by using the copula function $C(u_1,u_2)$ [17]:

$$H(x_1, x_2) = C(F_1(x_1), F_2(x_2)) = C(u_1, u_2) \tag{1}$$

The joint probability distribution of the two random variables can be characterised by a copula function in terms of their marginal probability distributions. An important implication of this theorem is that marginal modelling and dependence modelling can be carried out separately.

For given data X_1 and X_2, their dependence can be characterised by the empirical copula $C^{E}(u_1,u_2)$:

$$C^{E}(u_1, u_2) = \frac{1}{N}\sum_{m=1}^{N} I\left(\frac{\mathrm{rank}(x_{1,m})}{N+1} \leqslant u_1, \frac{\mathrm{rank}(x_{2,m})}{N+1} \leqslant u_2\right) \tag{2}$$

where N is the total number of data, $I(\bullet)$ represents the indicator function, and rank($x_{1,m}$) (or rank($x_{2,m}$)) is the rank of $x_{1,m}$ (or $x_{2,m}$) among x_1 (or x_2) in an ascending order. The empirical copula is a non-parametric description of dependence for a pair of random variables, which can be used for fitting various copula functions to data. A dependence measure that is suitable for copula modelling is the Kendall's τ coefficient:

$$\tau(X_1, X_2) = P\left[(X_1 - \widetilde{X}_1)(X_2 - \widetilde{X}_2) > 0\right] - P\left[(X_1 - \widetilde{X}_1)(X_2 - \widetilde{X}_2) < 0\right] \tag{3}$$

where $(\widetilde{X}_1, \widetilde{X}_2)$ is an independent copy of (X_1,X_2). The Kendall's τ measure is rank-dependent and invariant under strictly monotonic transformation.

In dealing with multi-variate data, the use of the normal and t copulas within a class of the elliptical copulas is popular. The bi-variate normal copula with the linear correlation coefficient ρ, $C_\rho^{N}(u_1,u_2)$, is given by:

$$\begin{aligned} C_\rho^{N}(u_1, u_2) &= \Phi_\rho(\Phi^{-1}(u_1), \Phi^{-1}(u_2)) \\ &= \int_{-\infty}^{\Phi^{-1}(u_1)} \int_{-\infty}^{\Phi^{-1}(u_2)} \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(-\frac{s^2 - 2\rho st + t^2}{2(1-\rho^2)}\right) \mathrm{d}s\mathrm{d}t \end{aligned} \tag{4}$$

where $\Phi_\rho(\bullet)$ is the bi-variate standard normal distribution with ρ, and $\Phi^{-1}(\bullet)$ is the inverse standard normal distribution. The bi-variate t copula with ρ and the degree-of-freedom parameter v, $C_{\rho,v}^{t}(u_1,u_2)$, is given by:

$$\begin{aligned} C_{\rho,v}^{t}(u_1, u_2) &= t_{\rho,v}(t_v^{-1}(u_1), t_v^{-1}(u_2)) = \int_{-\infty}^{t_v^{-1}(u_1)} \int_{-\infty}^{t_v^{-1}(u_2)} \\ &\quad \times \frac{1}{2\pi\sqrt{1-\rho^2}} \left(1 + \frac{s^2 - 2\rho st + t^2}{v(1-\rho^2)}\right)^{-(v+2)/2} \mathrm{d}s\mathrm{d}t \end{aligned} \tag{5}$$

where $t_{\rho,v}(\bullet)$ is the bi-variate t distribution with ρ and v, and $t_v^{-1}(\bullet)$ is the inverse t distribution with v. Both normal and t copulas are symmetrical, and the normal copula is a limiting case of the t copula when v becomes infinity. The advantage of the t copula is that it can capture lower and upper tail dependence of data (i.e. joint non-exceedance and exceedance probabilities for rare events). For the t copula, the other parameter v can be obtained by maximising the log-likelihood function.

Another widely-used copula family is the Archimedean copula. Popular Archimedean copulas include the Gumbel, Frank, and Clayton copulas, whose copula functions are given by:

$$C_\theta(u_1, u_2) = \exp\left(-\left[(-\ln u_1)^\theta + (-\ln u_2)^\theta\right]^{1/\theta}\right), \ \theta \geq 1 \tag{6}$$