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Static and Dynamic Stability of a Multi-stepped Timoshenko Column Including Self-weight

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ABSTRACT

A simplified and easy to implement approach for the solution of the static and dynamic stability of a Timoshenko column with multiple partitions is derived in a classic and condensed manner. The proposed methodology includes a non-uniformly distributed axial load along the structural member which can represent the self-weight of columns, chimneys, tall buildings or the axial load on piles induced by downdrag forces. The proposed model includes the effects of shear deformations along the member and the second-order shear force induced by the applied axial load as the member deforms. The effects of the self-weight on the structural member stability are studied using the proposed approach and conclusions regarding the contribution of the self-weight and discrepancies by neglecting it are presented. Three comprehensive examples including columns with weakened sections, distributed axial loads arising from the self-weight, symmetrical tapered sections, and stepped cross sectional members are included to validate and show the applicability of the proposed formulation.

1. Introduction

Generally, the effects of the self-weight are commonly neglected in the static and dynamic analyses of columns subjected to axial forces at the ends only. However, the behavior of tall buildings, chimneys, tapered structures, heavy and slender columns is typically affected by a combination of axial/self-weight and lateral loading interactions. Consequently, traditional stability approaches tend to overestimate critical buckling loads and erroneously predict the natural vibration frequencies. Axial distributed loads arise not only from the weight of column, but also from the loads carried by other attachments, lateral supporting members attached to the column, and other dead or live loads traveling through the column to the foundation level. These loadings produce non-uniform distributed and concentrated axial compression at various locations along the centroidal axis of the column. Top chord members of trusses can display similar loading conditions caused by axial forces transmitted from other connecting members. Other applications in Civil Engineering include members of variable cross-section and hence variable axial loading due to self-weight, or piles under compression caused by downdrag forces as a result of consolidation settlement of adjacent soils. The main objective of this paper is to provide a simplified solution of easy implementation in the analysis of Timoshenko columns subjected to axial load.

It is well-known that the governing differential equation of a column of variable cross section or subjected to distributed axial loading cannot be expressed in terms of constant coefficients [1]. Solutions to this problem have been provided in a closed-form only when the bending deformations are small compared to those due to shear loads (i.e., shear beam-column elements). For example, Aristizabal-Ochoa [2,3] and Hernandez-Urrea et al. [4] introduced exact and closed-form solutions for the analysis of shear beam-columns with semirigid end connections considering the member self-weight and lumped masses at the ends. Several formulations of this problem have been reported but based on complex and tedious mathematical methods to solve the governing differential equation. Timoshenko and Gere [1] and Vaziri and Xie [5] presented solutions of classical cases of tapered columns using Bessel functions or transforming a cantilever tapered column into a boundary value problem solved by numerical integration. These researchers concluded that the interaction between the critical end axial forces and the critical distributed axial loads are linear. Catellani and Elishakoff [6] developed a closed-form solution for a simply-supported column subjected to different types of distributed axial load using auxiliary problems and integral methods. Duan and Wang [7] introduced a mathematical method based on hyper-geometric functions

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to find the buckling loads for different classic column cases including the effects of self-weight. Their work showed that the calculated buckling loads are highly dependent on the boundary conditions. Additionally, Darbandi et al. [8] presented a model of an Euler-Bernoulli beam-column with elastic boundary conditions subjected simultaneously to transverse and longitudinal axial loadings treating the column as a singular perturbation boundary value problem. A common denominator of the available solutions is that the authors focus their attention on the static analysis only, neglecting the effects of shear deformations and forces, and also ignoring the principle of preserving angular momentum.

An alternative to tackle this problem is using *the transfer matrix method*. This method allows the use of a closed-form analytical solution in which the coupled effects of axial load, angular momentum, and bending and shear deformations can be accounted for explicitly in the formulation of the governing differential equation. The formulation can also include semirigid end connections, lateral bracings, and weakened sections on the static or dynamic analysis of tapered or stepped columns including their self-weight. This approach can also be used for the second-order analyses of columns with the aforementioned characteristics. In the past, Li [9–13] used this method to study the static and free vibration analyses of prismatic columns considering an indefinite number of weakened sections and lateral bracings. Takahashi [14] studied the free-vibration and stability behavior of non-uniform cracked Timoshenko columns using a transfer matrix in terms of a coupled set of first-order differential equations. Later Li [15,16] presented an improved formulation for the static stability analyses of a non-uniform column subject to distributed axial loads with an arbitrary number of cracks and boundary conditions.

The main objective of this publication is to propose a dynamic transfer matrix formulation capable of including the aforementioned coupled effects in addition to those caused by variable cross sections along the column with generalized boundary conditions of including shear and bending deformations. The proposed formulation will provide the possibility to study the static and dynamic behavior of tapered and stepped columns subject to concentrated and distributed axial loads including also arbitrary number of weakened sections and lateral bracings. Furthermore, the proposed dynamic transfer matrix is capable of accurately and efficiently predicting the nonlinear effects of tapered columns including its own weight under static and dynamic loads. Three comprehensive examples are presented in detail that show the efficiency and accuracy of this formulation, which at a reasonable computation effort, compares well with other theoretical methods, finite element models and experimental results.

2. Structural model

Consider the column AB shown in Fig. 1(a) made of n-steps with generalized end conditions. Each step is divided into m-segments due to the presence of $m-1$ notches. A typical step within the entire column is shown in Fig. 1(b) which is subdivided in several sub-steps. The steps are separated with lumped flexural springs of stiffness κ_{ij} (where i and j represent the step and a segment within the step, respectively). To study the beneficial effects of lateral bracings or for example to model the confinement provided by a soil mass around a weakened cross section of a driven pile, lateral springs of stiffness S_{ij}, were included in the analytical formulation. The rotational weakened indices κ_{ij} are normalized with respect to the bending stiffness of the particular segment with the ratios $R_{ij}=\kappa_{ij}/(E_iI_i/L_i)$, which are denoted as the b*ending stiffness indices* of the rotational springs. These ratios allow the simulation of any bending stiffness decay at any given sections of the column ranging from $R_{ij}=0$ to $R_{ij}=\infty$ for perfectly hinged and continuous sections, respectively. Furthermore, the lateral bracings are normalized with respect to the shear stiffness of the particular segment with the ratios $\overline{S}_{ij}=S_{ij}/(A_{s,i}G_i/L_i)$ which are denoted as the *shear stiffness indices* of the lateral bracings. These ratios may vary from $\overline{S}_{ij}=0$ to ∞ for unbraced members (or zero confinement provided at the weakened section) and for perfectly braced structural members, respectively.

In the proposed formulation, it is assumed that each step of the structural member is made of a homogenous linear elastic material with Young and shear moduli E_i and G_i, respectively. The centroidal axis of each column segment is to be assumed a straight line collinear with the previous and subsequent segments, and axially loaded at its ends along its centroidal x-axis with a constant load P_i (tension or compression). The cross section of any column segment is to be assumed doubly symmetric (which implies that its centroid coincides with its shear center) and described by the gross area, A_i, effective shear area, A_{si}, and principal second moment of area, I_i, about the plane of bending. In the proposed approach, all transverse deflections, rotations, and strains along the column are assumed to be relatively small, so that the principle of superposition can be applied. Regarding the weakened sections and lateral bracings or confinement, it is assumed that these sections are perpendicular to the straight axis of the column as a whole and that both the moment-rotation and force-deflection behaviors are perfectly linear-elastic.

2.1. Governing equations

The static and dynamic behavior of the jth-segment of the multi-weakened column is governed by the following equilibrium equations at the differential element level shown in Fig. 1(c). The equations of equilibrium (1) and (2) are expressed in terms of dimensionless parameters $\overline{m}_i$ and r_i (defined as the mass per unit length of the beam-column and radius of gyration of its cross section; respectively).

$$\frac{\partial V_{i,j}}{\partial x_{i,j}}=-\overline{m}_i\frac{\partial^2 y_{i,j}}{\partial t^2} \tag{1}$$

$$\frac{\partial M_{i,j}}{\partial x_{i,j}}=V_{i,j}+\overline{m}_i r_i^2\frac{\partial^2\theta_{i,j}}{\partial t^2}-\left(P_i+\sum_{k=i+1}^{n}P_k\right)\frac{\partial y_{i,j}}{\partial x_{i,j}}. \tag{2}$$

Assuming that E_i, G_i, A_i, A_{si}, $\overline{m}_i$ and P_i remain constant along step i and that the applied axial loads induce additional shear forces, the following relationships can be written:

$$V_{i,j}=A_sG\gamma_{i,j}+P\theta_{i,j}, \tag{3}$$

$$M_{i,j}=E_iI_i\frac{\partial\theta_{i,j}}{\partial x_{i,j}}, \tag{4}$$

$$\gamma_{i,j}=\theta_{i,j}-\frac{\partial y_{i,j}}{\partial x_{i,j}}, \tag{5}$$

Since Eqs. (1) and (2) are coupled together, they can be reduced to a single fourth-order differential equation in terms of the dimensionless parameters listed in Table 1 as follows:

$$\frac{d^4\overline{Y}_{i,j}}{d\overline{x}_{i,j}^4}+2\Omega_i\frac{d^2\overline{Y}_{i,j}}{d\overline{x}_{i,j}^2}+\varepsilon_i\overline{Y}_{i,j}=0 \tag{6}$$

where:

$$2\Omega_i=b_i^2s_i^2+b_i^2R_i^2+F_i^2+\left(F_i^4+\sum_{t=i+1}^{n}F_t^2s_t^2\right)s_i^2, \tag{7}$$

and

$$\varepsilon_i=b_i^4R_i^2s_i^2-b_i^2\left(1+F_i^2s_i^2+\sum_{t=i+1}^{n}F_t^2s_t^2\right). \tag{8}$$

An advantage of the proposed incremental stepped formulation presented herein is that each variable is constant and thus, it does not require complex mathematical formulations for its solution. This is achieved by applying concentrated axial loads at the end of each segment that if accumulated, could be used to study the static and dynamic