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An Analytical Failure Envelope for the Design of Textile Reinforced Concrete Shells☆

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ABSTRACT

Shells have the potential to considerably reduce material consumption in buildings due to their high structural efficiency compared to equivalent structures acting in bending. Textile reinforced concrete (TRC) is a promising material for the construction of thin concrete shells due to its strength, geometric versatility, and durability. Existing design methods for TRC shells predicts the local capacity by linear interpolation between experimentally determined values of strength in pure tension, pure bending, and pure compression. This simplification leads to a significant underestimation of strength in combined bending and compression. Relying entirely on physical test results also effectively prohibits exploration and optimisation of the shell design. This paper proposes a new analytical design approach for TRC which is instead derived from the properties of the concrete and reinforcement, and for the first time captures the highly non-linear interaction between axial and bending forces.

A series of pure tension, pure bending, and combined bending and compression tests were carried out on TRC specimens of 15 mm and 30 mm thickness. The predicted strengths were conservative under combined compression and bending but otherwise accurate. For the specimens tested, the proposed method increases the predicted strength by a factor of up to 3.7 compared to existing methods, whilst remaining conservative, and hence its use could lead to significant material savings and new applications for TRC shells.

1. Introduction

Thin compression shells have long been a means of creating large spans, from historic masonry domes and barrel vaults to the famous 20th century reinforced concrete shells by Torroja, Candela, Nervi and Isler [1]. More recently, renewed interest in shells is being driven by modern advances in computational design, automated manufacturing and construction materials, as well as sustainability concerns [2]. The high structural efficiency of shells creates the potential for significant material and weight savings when compared to bending structures of equivalent strength and can facilitate the use of low strength materials with lower associated carbon emissions [3, 4].

Whilst it is possible in theory to design a shell to act purely in compression, in practice bending and tensile forces arise due to geometric constraints, temporary construction loading states, settlement of foundations, accidental damage and variable live loadings. Textile reinforced concrete (TRC) is a composite material consisting of fine-grained concrete and layers of woven textile (usually of glass or carbon fibres), which gives the material bending and tensile strength. The flexibility of the reinforcement and absence of cover requirements for durability allows the practical construction of thin shells with complex geometries. A growing number of projects are now being realised, including footbridges, cladding panels and roof canopies [5, 6].

The behaviour of TRC is non-linear and anisotropic, due to cracking of the concrete and subsequent reinforcement crack-bridging and debonding. Stresses and deformations in TRC can however be modelled using a microplane damage model as proposed by Chudoba et al. [7]. This can also be used to predict failure [8], however for strength design with multiple loadcases it is more practical to calculate forces using a linear analysis. Thin TRC sections fail under certain combinations of axial forces and bending moments, and their strength can therefore be described using a failure envelope plotted on an axial-moment interaction diagram. This approach is similar to that used in the design of reinforced concrete columns.

Historically, research into the structural performance of TRC has focused on tensile behaviour [9, 10] as tensile capacity is critical in many applications of TRC, such as strengthening of existing structures [11, 12], anticlastic shells [6, 13] and thin-walled beams [14, 15]. Scholzen et al. [16] propose a bi-linear failure envelope defined by linear interpolation between three experimentally determined

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strengths, one in pure compression, one in pure tension and one in pure bending. The bi-linear approximation under tensile loading has been verified experimentally [17]. The linear approximation in compression is conservative but this has been shown to be acceptable for the tension-critical structures for which the method has so far been employed [6, 8]. However, in well-conditioned compression shells, tensile forces are typically much smaller than compressive forces, or not present at all, and the compressive region of the failure envelope is of greatest interest. The failure envelope of steel reinforced columns under combined bending and compression is well understood to be non-linear, and this is also the case for columns with glass fibre reinforced polymer (FRP) reinforcement [18]. A simple extension to the bi-linear envelope was proposed by Hawkins et al. [4], where the addition of a fourth data point (corresponding to a triangular concrete stress distribution) creates a tri-linear envelope. However, it is proposed here that a more realistic model be created to further improve design efficiency and describe the behaviour of TRC more accurately. Furthermore, since current failure envelopes [4, 16] rely on experimentally determined strength values of individual TRC sections, the extent to which the designer can quickly explore possible variations in section thickness or reinforcement layout is limited.

This paper introduces an analytical model of TRC strength based on the stress-strain relationships of the constituent materials. This enables multiple sections to be analysed from a single set of tests, and captures the non-linear interaction between axial and bending forces causing failure.

2. Materials

A series of TRC specimens were constructed and tested out to determine their strength under combinations of axial and bending loads. This section describes the concrete and textile reinforcement used.

2.1. Fine-grained concrete

A fine-grained concrete mix was developed with the aim of creating workable material using readily available components, with a target strength at 28 days of 50 MPa. The proportion of Portland cement was kept to a minimum to lessen the embodied CO_2 of the mix and lower the alkalinity, which is shown to reduce the time-dependant strength degradation of alkali-resistant (AR) glass fibre reinforcement [19]. The final mix composition is shown in Table 1. The binder is made up of 70% Portland cement and 30% fly ash (conforming to BS EN 450 N [20]), the water to binder ratio is 0.4 and the aggregate to binder ratio is 3.0. 10 ml of polycarboxylate superplasticiser was added per kg of binder.

A maximum aggregate size of 2 mm was used to enable construction of thin cover layers and penetration of reinforcement mesh. The particle size distribution of the aggregate was found to be of critical importance in achieving the target strength. It was found in preliminary testing that reducing the ratio of 0–1 mm particles to 1–2 mm particles from the natural ratio of 3:1 to 1:1 increased the compressive strength by 43%. This equal ratio was used in the final mix. The measured density of the material is 2197 kg/m^3 (at 28 days).

Four 160 × 40 × 40 mm prisms were tested to determine the strength and stress-strain relationship. Each prism was loaded along its long axis in a concrete compression testing rig after 28 days curing in a water bath at room temperature. Strain in the specimen was measured using a pair of extensometers on opposite sides of the specimen, measuring displacement over a gauge length of 80 mm. The average strain measured from each test is plotted in Fig. 1. The small loops in the data at lower stresses were caused by rapid fluctuation in the oil pressure of the rig at the start of the test, and can be ignored. The average strength was 47.2 MPa, reached at an average peak strain of 0.192%.

Table 1
Fine-grained concrete composition.

Portland cement	349 kg/m^3
Fly ash	150 kg/m^3
Aggregate (0–1 mm)	747 kg/m^3
Aggregate (1–2 mm)	747 kg/m^3
Water	199 kg/m^3
Superplasticiser	4982 ml/m^3

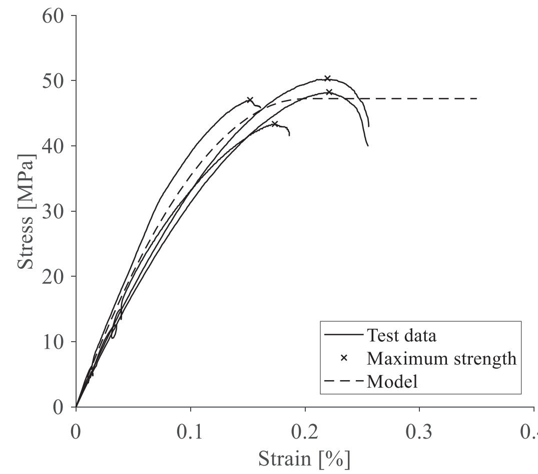

Fig. 1. Fine-grained concrete prism test results.

A parabola-rectangle approximation (as described in the FIB Model Code [21] and BS EN 1992-1-1 [22]) is also plotted in Fig. 1. This model is not a reproduction of an experimentally determined stress-strain curve but is used as a simplification of more complex behaviour under three-dimensional stress states in concrete beams [23]. The curve is defined by the design compressive strength (f_{cd}) as well as three other parameters; the strain at peak strength (ε_{c2}), the strain at failure (ε_{cu2}) and the exponent (n), as in Eq. 1.

$$\begin{aligned}\sigma_c &= f_{cd}\left(1-\left(1-\frac{\varepsilon_c}{\varepsilon_{c2}}\right)^n\right) && \text{for} \quad 0 \le \varepsilon_c \le \varepsilon_{c2} \\ &= f_{cd} && \text{for} \quad \varepsilon_{c2} \le \varepsilon_c \le \varepsilon_{cu2}\end{aligned} \tag{1}$$

For concrete with characteristic strength below 50 MPa, these values are typically assumed to be $\varepsilon_{c2} = 0.2\%$, $\varepsilon_{cu2} = 0.35\%$ and $n = 2$ (as plotted in Fig. 1). Fine-grained concretes used for TRC are often less stiff than typical concrete of equivalent strength due in part to a lower proportion of aggregates. Strains at peak strength of up to $\varepsilon_{c2} = 0.5\%$ have been reported [16, 24, 25], however in this case the typical stiffness values fit the data well. This model was therefore adopted in further analysis.

2.2. Reinforcement

The reinforcement material is an AR-glass fibre textile with acrylic resin coating, chosen due to its wide availability, affordability and flexibility for the formation of curved shell structures. The yarns in the warp direction consist of straight bundles of fibres, whilst in the fill direction the yarns are in groups of three and are woven between the warp yarns. Individual yarns in both directions have a similar weight, but the variable spacing leads to different reinforcement areas per unit length. Key properties of the material are shown in Table 2. The area in each direction was calculated based on an assumed density of 2700 kg/m^3.

Tensile tests on eight warp and eight fill yarns were carried out to determine the ultimate strength (f_t) and stiffness (E_t). The strain was measured using a laser extensometer, with the test set-up shown in Fig. 2. The test results showed brittle-elastic failure. In each test, failure occurred at the interface between the yarn and anchor. Despite having