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Theoretical and Numerical Study on the Natural Frequencies of Bridges With Corrugated Steel Webs

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ABSTRACT

The dynamic characteristics including natural frequencies, mode shapes, and damping ratios are essential to investigate the vibration response of a structural system, in which the natural frequency is deemed the most important parameter. Research on the vertical natural frequencies of prestressed concrete (PC) box girders with corrugated steel webs (CSWs) is limited. In this research, the energy variation method proposed by E. Reissner and Galerkin method was adopted to study the single-span and multi-span continuous prismatic PC box girders with CSWs and simply-supported condition. To the accurately derive the formula for the vertical natural frequencies, the shear lag effect and shear effect from the box girder and CSWs, respectively, are considered. Numerical simulations were carried out to validate the theoretical solution. Sensitivity studies using the analytical equation were also conducted to investigate the effects of the waveform of CSWs, the outer top flange, and the thickness of corrugated steel webs.

1. Introduction

With the high shear-buckling resistance and out-of-plane bending stiffness, corrugated steel plates (Fig. 1) have been widely used as offshore, building, and bridge components, e.g., blast walls [1], CWC-BRB [2], box girder webs [3], and so on. Since the construction of Cognac Bridge in France in 1986 [4], the use of corrugated steel webs (CSWs) in prestressed concrete (PC) bridges has gained greater popularity. Compared with the PC box girder with conventional flat webs, the CSWs carry only shear forces [5], the girder's flanges and concrete slab carry the axial load and bending moment [6], the deadweight of box girder is greatly reduced, and the efficiency of prestressing is enhanced.

PC bridges with CSWs have been studied by several researchers. Ibrahima et al. [7] investigated the behavior of plate girders with trapezoidal CSWs under fatigue loading and developed a relationship between the stress range and the number of loading cycles for failure. He et al. [8] analyzed the deflection and mechanical property based on the elastic bending theory considering shear deformations. Abbas et al. [9], Eldib [10], and Hassanein et al. [11] investigated the shear buckling behavior of box girders with corrugated web plates. Wang et al. [12] examined the fatigue strength of the CSW girders with several welded structural details and welding methods experimentally. Nie et al. [13] proposed an innovative optimization scheme for both positive and negative moment regions to improve the global performance of a continuous composite PC girder with CSWs. Mo et al. [14] described the experimental results of four scaled PC box-girder bridges with CSWs subjected to torsion and presented an analytical model to predict the torsional behavior of such bridges. Chen et al. [15] reported the long-term behavior of PC bridges with CSWs and proposed a numerical model considering the creep and shrinkage of concrete and tendon relaxation. Ren et al. [16] derived the theoretical calculation method for the torsional vibration of concrete box girders with CSWs where the influence of the diaphragm set on the torsional vibration of the concrete box girder was considered. Manko and Beben [17] presented the results (speed, dynamic coefficients, velocity vibration, and vibration frequency) based on the dynamic load tests of a corrugated steel arch bridge. Ji et al. [6] calculated the natural vibration frequencies for a two-span continuous box girder with CSWs. Liu et al. [18] calculated and analyzed the dynamic characteristics of the girders with CSWs experimentally and numerically. Sung et al. [19] studied the static and dynamic characteristics of the bridges with CSWs using both experimental and numerical methods to develop a bridge monitoring system. An et al. [20] obtained the theoretical restrained torsion frequencies for the composite box girder with CSWs, according to the theory of Umansky.

With the rapid advancement in material and construction technology, designing multi-span continuous PC box girders with CSWs has become a trendy development in the structural engineering world. The

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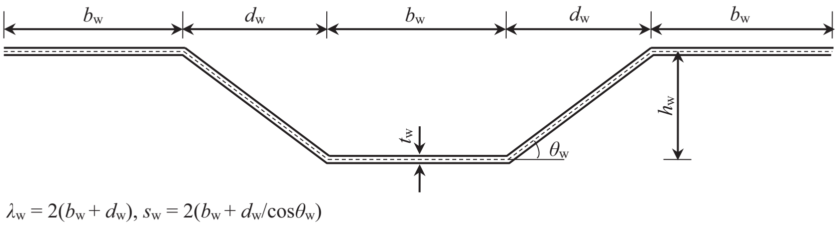

Fig. 1. The schematic diagram of corrugated steel web.

dynamic characteristics such as natural frequencies, mode shapes, and damping ratios are essential to investigate the vibration response of a structural system. Consequently, accurate evaluation on the dynamic performance is an important issue. For a vibration analysis, the natural frequency is regarded as the most important parameter. Research on the simple calculation formulation on the vertical natural frequencies of PC box girders with CSWs is limited. The main object of this paper is to deal with this issues. The energy variation method proposed by E. Reissner and Galerkin method was adopted in this study to establish and solve the governing equations of the single-span and multi-span continuous PC box girders with CSWs, prismatic sections, and simply-support edges. To derive the accurate formula for the vertical natural frequencies, the shear lag effect and shear effect from the box girder and the CSWs, respectively, are considered. Numerical simulations were made to validate the theoretical solution. Sensitivity studies using the analytical equation were also conducted to investigate the effects of the waveform of CSWs, the outer top flange, and the thickness of CSWs.

2. Warping displacement function

The single thin-walled box girder (Fig. 2) has been most commonly used. For this type of box girders, the variational method [21] is an effective method to analyze the influence of shear lag effects on the vertical natural frequencies.

For the free bending vibration of a PC box girder with CSWs, the deflections at the top and bottom flange plane do not exactly satisfy the cross-section assumption because of the shear lag effect. So, applying only one generalized displacement (e.g., beam deflection $\omega(x,t)$) to describe the bending vibration of such girders is deemed inappropriate. Instead, the warping displacement with the shear lag effect should be considered. In this paper, the proposed warping displacement function is as follows

$$v(x,y,z,t) = g(y)u(x,t) - z\varphi(x,t) \tag{1}$$

$$g(y) = \begin{cases} a_1 \cos\frac{\pi y}{2b_1} + D & \text{Bottom flange} \\ -\cos\frac{\pi y}{2b_2} + D & \text{Inner top flange} \\ -a_3 \cos\frac{\pi(y - b_2 - b_3)}{2b_3} + D & \text{Outer top flange} \end{cases} \tag{2}$$

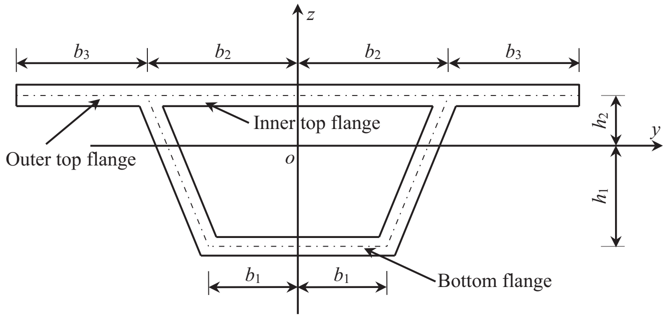

Fig. 2. The cross section of a single thin-walled box girder.

$$a_1 = \frac{h_1 b_1^2[8b_2^2(1+\mu) + l^2]}{h_2 b_2^2[8b_1^2(1+\mu) + l^2]},\quad a_3 = \frac{b_3^2[8b_2^2(1+\mu) + l^2]}{b_2^2[8b_3^2(1+\mu) + l^2]},\quad D = \frac{2(A_2 + a_3A_3 - a_1A_1)}{\pi A} \tag{3}$$

where $g(y)$ is the warping displacement function; x, y, and z are the coordinates; t is the time; $u(x, t)$ is the displacement amplitude of warping displacement function; $\varphi(x, t)$ is the angular displacement due to bending deformation; μ is the Poisson ratio; A is the cross sectional area neglecting the web's area; A_1, A_2, and A_3 are respectively the sectional areas of bottom flange, inner top flange, and outer top flange. a_1, a_2, and a_3 are the constants; and b_1, b_2, and b_3 are also the constants.

3. Governing equation

Based on the above warping displacement function, the total strain energy in the top and bottom flanges is

$$\begin{aligned} V_F &= \frac{1}{2}\int_0^l \int_A E\varepsilon^2 dxdA + \frac{1}{2}\int_0^l \int_A G\gamma^2 dxdA \\ &= \frac{1}{2}\int_0^l \int_A E\left[g(y)\frac{\partial u}{\partial x} - z\frac{\partial \varphi}{\partial x}\right]^2 dxdA + \frac{1}{2}\int_0^l \int_A G\left[\frac{dg(y)}{dy}u\right]^2 dxdA \\ &= \frac{1}{2}\int_0^l E\left(\frac{\partial \varphi}{\partial x}\right)^2 dx \int_A z^2 dA - \int_0^l E\frac{\partial u}{\partial x}\frac{\partial \varphi}{\partial x}dx \int_A zg(y)dA \\ &+ \frac{1}{2}\int_0^l E\left(\frac{\partial u}{\partial x}\right)^2 dx \int_A [g(y)]^2 dA + \frac{1}{2}\int_0^l Gu^2 dx \int_A \left[\frac{dg(y)}{dy}\right]^2 dA \\ &= \frac{1}{2}\int_0^l EI\left(\frac{\partial \varphi}{\partial x}\right)^2 dx - \int_0^l EC_1\frac{\partial u}{\partial x}\frac{\partial \varphi}{\partial x}dx + \frac{1}{2}\int_0^l EC_2\left(\frac{\partial u}{\partial x}\right)^2 dx \\ &\quad + \frac{1}{2}\int_0^l GC_3 u^2 dx \end{aligned} \tag{4}$$

where

$$I = \int_A z^2 dA \tag{5}$$

$$C_1 = \int_A zg(y)dA = -h_1A_1\left(D + \frac{2a_1}{\pi}\right) + h_2A_2\left(D - \frac{2}{\pi}\right) + h_2A_3\left(D - \frac{2a_3}{\pi}\right) \tag{6}$$

$$C_2 = \int_A [g(y)]^2 dA = A_1\left(D^2 + \frac{4Da_1}{\pi} + \frac{a_1^2}{2}\right) + A_2\left(D^2 - \frac{4D}{\pi} + \frac{1}{2}\right) + A_3\left(D^2 - \frac{4Da_3}{\pi} + \frac{a_3^2}{2}\right) \tag{7}$$

$$C_3 = \int_A \left[\frac{dg(y)}{dy}\right]^2 dA = \frac{\pi^2}{8}\left(\frac{A_1a_1^2}{b_1^2} + \frac{A_2}{b_2^2} + \frac{A_3a_3^2}{b_3^2}\right) \tag{8}$$

and E is elasticity modulus of concrete.

The shearing strain of a PC box girder with CSWs is

$$V_W = \frac{1}{2}\int_0^l G_sA_s\left(\frac{\partial \omega}{\partial x} - \varphi\right)^2 dx \tag{9}$$

where G_s [27] is the equivalent shear modulus of the CSW; and A_s is the sectional area of the CSW.

So, the total strain energy for the PC box girder with CSWs is