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An Insight Into Structural Design Against Deflection

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ABSTRACT

There are “rules of thumb” for designing structural elements such as beams, columns and floors, which are simple and effective. Such rules are familiar to most engineers and are widely used to develop quick preliminary designs. Are there similar types of “rules of thumb” that can be used for conceptual designs to achieve smaller deformations of whole structures or make such structures more effective, efficient and elegant? This article demonstrates four conceptual solutions to reduce deflections of structures which are all related to internal forces. Examples are provided to illustrate the use and effectiveness of the concepts, including some well-known structures.

1. Introduction

For preliminary design of elements, structural engineers have often used “rules of thumb” e.g. maximum span to depth ratios for beams. Following preliminary sizing, the element design is refined according to the requirements of the associated national design code etc. In developing whole structural form, consideration of the overall stiffness of the structural system is important. The deflection of structures is a key serviceability consideration and may often control design. As buildings become taller and spans increase for bridges, roofs etc., the associated deflections of these structures become a major design issue: the question then arises of how best to design structures against deflections? This article explores the potential of harnessing simple approaches akin to the ‘rules of thumb’ common in element design, to produce efficient whole-structure conceptual designs in regards to deflection.

At structural element level, many of the current design approaches are based on the understanding of a simple equation for beams often presented in textbooks as follows [1,2]:

$$\Delta_{\max} = \alpha \frac{qL^4}{EI} \tag{1}$$

where $\Delta_{\max}$ is the maximum deflection, q is a distributed load, L is the span, E modulus of elasticity and I the second moment of area of the cross-section of the beam. The parameter α is a non-dimensional coefficient relating to boundary conditions, for example, 5/384 for a simply supported beam and 1/8 for a cantilever when q is a uniformly distributed load. Implementation of Eq. (1) for reducing deflection has been seen in practice as outlined below following a “rule of thumb” format:

a) *Reducing span L:* As the deflection is proportional to L to the power of four, reducing span where possible is the most effective way to reduce deflection e.g. via provision of additional supports.
b) *Increasing second moment of area I*: This is normally applicable to local members, such as using a larger cross-section or adding material as far away as possible from the neutral axis of a given cross-section to enlarge the I value effectively. Conceptually, a tall building can be seen as a huge cantilever, the large second moment of area of its cross-section can be achieved by reasonably arranging the positions of columns and shear walls of the building.
c) Reducing α: This can be achieved by enhancing the boundary conditions, such as changing pinned supports to fixed supports. Alternatively, adding elastic supports to a structure is often adopted. For example, the cables of a cable-stayed bridge provide elastic supports to the deck, allowing long spans; in this case the bridge deck can be seen as a beam on an elastic foundation.

At whole structure level, the maximum displacements of any pin-connected structure and rigid frame structure are shown respectively in Eqs. (2) and (3) [1,2]:

$$\Delta_{\max} = \sum_{i=1}^{s} \frac{N_i \overline{N}_i L_i}{E_i A_i} \tag{2}$$

$$\Delta_{\max} = \sum_{i=1}^{s} \frac{\int_0^{L_i} M_i(x) \overline{M}_i(x) dx}{E_i I_i} \tag{3}$$

where N_i and $\overline{N}_i$ are the axial forces of the ith member induced by the actual loads and that by a unit load applied at the critical point where

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the maximum displacement is likely to occur respectively; $M_i(x)$ and $\overline{M}_i(x)$, similar to N_i and $\overline{N}_i$, are the bending moment of the ith member induced by the actual loads and by a unit load at the critical point respectively. L_i, E_i, A_i and I_i ($i = 1, 2, \ldots, s$) are the length, Young's modulus, area and second moment of area of the cross-section of the ith member respectively.

Eqs. (2) and (3) provide a method for calculating the deflection of any structures with pinned or rigid connections. Eq. (2) is suitable for trusses, scaffoldings and lattice structures, and has a history of over 150 years [3]. However, Eq. (2) is often given a lack of emphasis in many current textbooks on Mechanics of Materials and Structural Analysis. This is because it requires obtaining the internal forces N_i and $\overline{N}_i$ before calculating the displacement, such a calculation may be regarded as too tedious. Normally very simple statically determinate plane trusses are provided to show how Eq. (2) is used [1,2]. In difference to Eq. (1), implementation of Eqs. (2) and (3) for reducing the maximum displacement of a whole structure is not well known. This article demonstrates four rules of thumb for whole structure behaviour which are analogous to those described for Eq. (1). These are intuitively interpreted based on Eqs. (2) and (3) and application of each rule of thumb is illustrated by examples in sections 3 to 6.

2. Theoretical basis for designing structures against deflection

It is difficult to identify the physical essence of Eqs. (2) and (3). This is because N_i and $M_i(x)$ ($i = 1, 2, \ldots, n$) are functions of loading that can have many variations. If we consider the worst loading scenario whereby all the loads on the structure are lumped at the critical point at which the largest deflection is likely to occur. Then the load is normalised to a unit. This loading is not true but the worst scenario for deflection is considered consistently for all structures to be discussed. The critical points of some structures can be easily identified. For example, for a horizontal cantilever, the critical point for a vertical load would be at the free end of the cantilever; for a simply supported rectangular plate, it would be at the centre of the plate for a vertical load; and for a plane frame supported at its base, it would be at the top of the frame for horizontal loading.

This treatment of loading removes any possible variation of loads and allows focus to be on the behaviour of the structures. For calculating the deflection at the critical point, the normalised unit load and the unit force act at the same critical point. Thus N_i becomes $\overline{N}_i$ and $M_i(x)$ becomes $\overline{M}_i(x)$ for the normalised unit load and Eqs. (2) and (3) can be represented as:

$$\overline{\Delta}_{\max} = \sum_{i=1}^{s} \frac{\overline{N}_i^2 L_i}{E_i A_i} \tag{4}$$

$$\overline{\Delta}_{\max} = \sum_{i=1}^{s} \frac{\int_0^{L_i} \overline{M}_i^2(x)dx}{E_i I_i} \tag{5}$$

The integration $\int_0^{L_i} \overline{M}_i^2(x)dx$ describes the area under the curve $\overline{M}_i^2(x)$, which in turn can be represented by the area of a rectangle with a length L and a mean height $\overline{M}_{m,i}^2$. Therefore, Eq. (5) can be equally expressed as:

$$\overline{\Delta}_{\max} = \sum_{i=1}^{s} \frac{\overline{M}_{m,i}^2 L}{E_i I_i} \tag{6}$$

Now Eqs. (4) and (6) have similar mathematical forms. The physical meaning of $\overline{\Delta}_{\max}$ in Eqs. (4) to (6) is the inverse of the static stiffness of a pinned structure or a rigid frame structure [4,5], and in a more general sense, $\overline{\Delta}_{\max}$ is the largest coefficient in a flexibility matrix, i.e. the inverse of the stiffness matrix, of any truss or beam type of structure [6]. Intuitively, if a structure has a larger static stiffness or smaller largest flexibility coefficient, the structure would have a smaller deflection. Instead of conducting any particular calculation, we can intuitively interpret Eq. (4) to capture the physical essence of the relationship, which can provide a basis for practical application.

It can be noted that all terms on the right side of Eq. (4) are positive. If L_i/E_iA_i does not change significantly for all members, from Eq. (4) we can make the following observations towards achieving smaller deflections [4,5]:

- For $\overline{\Delta}_{\max}$ to be smaller, the many positive terms on the right-side of Eq. (4) should be zero in a mathematical sense, in other words, many $\overline{N}_i$, i.e. the internal forces in the members, are zero. This corresponds to the physical scenario that the load acting at the most unfavourable location is transmitted to the supports of the structure without passing through these zero force members. In other words, the load goes through a more direct or shorter internal force path.
- The other way to make $\overline{\Delta}_{\max}$ smaller is to make all the positive items on the right-side of Eq. (4) smaller. This corresponds to the state of smaller internal forces.
- As the square of a larger term will contribute more significantly to $\overline{\Delta}_{\max}$ than other smaller terms on the right-side of Eq. (4), to achieve smaller $\overline{\Delta}_{\max}$, the value of the internal force, $|\overline{N}_i|$, of the ith member should not be much larger than other internal forces. This corresponds to the scenario that a structure should have more uniformly distributed internal forces to achieve smaller deflection.

These observations and interpretations can be represented in a more concise and memorable way in terms of structural concepts as follows [4,5]:

i. The more direct the internal force path, the smaller the deflection.
ii. The smaller the internal forces, the smaller the deflection.
iii. The more uniform the distribution of internal forces, the smaller the deflection.

The same can be obtained from Eq. (6). When internal forces of a structure contain bending moments and axial forces, both Eqs. (4) and (6) need to be used and the displacements are added up. It has been noted that the displacement derived from bending moments is often far larger than that from axial forces. Therefore, the fourth concept is

iv. The more bending moments being converted to axial forces, the smaller the deflection.

The above four statements are abstracted from Eqs. (4) and (6) with an artificial concentrated load rather than the actual loading in Eqs. (2) and (3) that are used to calculate the actual deflections at a given loading. Similar to Eq. (1), Eqs. (4) and (6) are used to draw qualitative conclusions and reveal the physical essence of structural behaviour.

Eq. (1) can also be represented as:

$$\Delta_{\max} = \alpha \frac{qL^4}{EI} = \alpha \frac{\beta M_{\max}^2}{EIq} \tag{7}$$

where β is a constant, for example, 64 for a simply supported beam and 4 for a cantilever when q is a uniformly distributed load. Eq. (7) shows qualitatively that reducing span is equivalent to reducing the maximum bending moment, which corresponds to the second concept/statement above. The significance of Eqs. (4) and (6) is that they represent whole structures rather than structural members and that the concepts deduced are applicable to all types of structure whose internal forces are represented by axial forces and bending moments.

The four structural concepts or statements all deal with internal forces and provide a fundamental basis for designing structures against deflections and for developing different physical measures to achieve desired internal forces that lead to smaller deflections and more efficient structures. The four statements are also reversible as they are abstracted from Eqs. (4) and (6). In other words, limiting deflection will lead to more direct internal force path, smaller internal forces, more