# Exploring Bridge Dynamics for Ultra-high-speed, Hyperloop, Trains

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## ABSTRACT

In this paper the dynamics of a set of ultra-high-speed (UHS) moving masses/loads traversing a continuous beam are explored. The proposed model is intended to simulate the dynamic response of continues bridges under the new Hyperloop/Transpod trains, which are proposed to travel at up to 1200 km/h. This speed introduces a range of dynamic responses that have hitherto not been observed in generic high-speed trains. The analytical results show that the dynamic amplification factors, due to train passage, are significantly larger than current trains. This is due to the combination of ultra-high-speed and continuous beam construction, which is necessary to maintain a partial vacuum in the enclosed tube. Therefore, current design recommendations are not sufficient for these UHS trains.

## 1. Introduction

The Hyperloop Alpha [1,2] and Transpod [3] are trains that travel at Ultra-High-Speeds (UHS trains). This UHS is achieved by having the train travel within a "vacuum" tube as shown in Fig. 1. Traveling within these tubes, which are continuous beams, allows a train to circumvent the air resistance, drag forces, of conventional high-speed trains. In addition, this UHS conceptual design makes use of magnetic levitation and linear accelerators as a means of propulsion. Thus, the concept is to reduce friction, in all its forms to an extremely low level. The proposed working speed of around 970 km/h, with a top speed of 1200 km/h has been suggested. This compares with an average working speed of 270 km/h for High-Speed trains (HS trains). The latest record for a conventional passenger train is held by an SNCF (France) TGV POS trainset, which reached 574.8 km/h (357.2 mph) in 2007. This speed was exceeded (in Japan on a national test track) by the unconventional seven-car L0 series trainset which attained a speed of 603 km/h (375 mph) in 2015.

The current state-of-practice for the design and assessment of railway bridges in the UK is comprehensively treated in [4]. This document considers the effect of impact, oscillation and track and wheel irregularities. It suggests that bridge dynamics only plays an important role at train speeds above 160 km/h. Therefore, [4] recommends that the dynamic amplification factor (DAF) is 1 for train speeds below 160 km/h. This has been confirmed by other researchers in the assessment of existing railway bridges in the UK using nonlinear analysis techniques [5,6]. Eurocode EN 1991–2 [7] has similar methods for calculation of the DAF for train speeds up to 200 km/h. However,

[7] suggests more rigorous dynamic analysis is required to calculate the DAF for train speeds more than 200 km/h. Thus, for conventional HS trains, there is a need to consider the dynamic amplification effects by more thorough and bridge/train specific analyses. Nevertheless, this dynamic amplification while important is still likely to be relatively small at speeds of 270 km/h.

The UHS trains could travel at speeds more than four times the average speed of HS trains. This would be double the current world record speed. At these speeds the importance of dynamic amplification may be significant. This raises the new, currently unsolved, question of what is an appropriate DAF for this case. Furthermore, the Hyperloop/Transpod tubes will be supported by a series of piers which constrain the tube in the vertical direction but allow longitudinal slip for thermal expansion as well as dampened lateral slip to reduce the risk posed by earthquakes. The spacing of the Hyperloop piers retaining the tube is critical to achieve the design objective of the tube structure. The average spacing is 30 m, which means there will be near 25,000 piers between the proposed San Francisco-Los Angeles line [1,2]. This imposes very large dynamic loading on the piers, which is currently not considered in any design standards. Therefore, exploring the impact of UHS train on the current DAF in the design standards is vital.

The mathematics of a moving force was first discussed by [8] and in the excellent and thorough treatise [9] that discusses both moving force and mass problems of simple spans. However, only a very limited class of simply moving load problems can be solved analytically. Thus, numerical methods are necessary for more general moving load simulations [10]. Authors [11] present a good historical review of the code based dynamic amplification factors (DAF) caused with traveling loads

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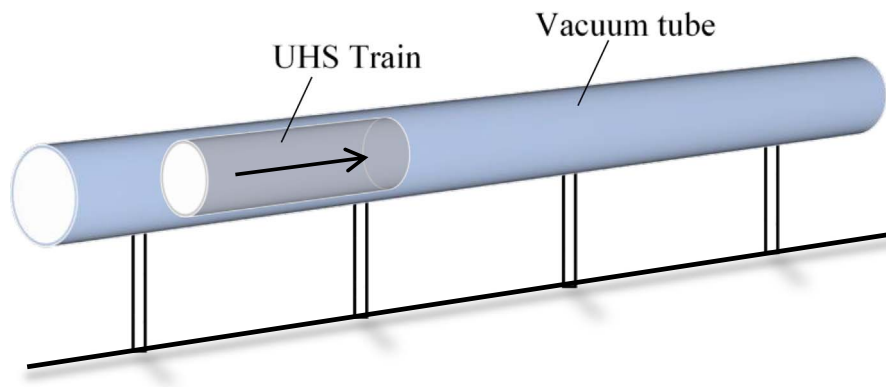*E-mail address:* nick.alexander@bristol.ac.uk (N.A. Alexander).

**Fig. 1.** Initial conceptual design of prototype Hyperloop one.

in the context of road traffic. The DAF represents the increase in quasi-static peak deflections and/or stress caused by the dynamics of the traveling load. A significant component of the DAF, in this case of highway traffic, is due to the impulsive "shock" loads of heavy vehicles traversing defects in the road surface. The true moving mass/load dynamic amplification is very minimal at the speeds of highway traffic. Therefore, larger values of DAF are observed in the cases of high speed trains. A similar concept is also considered in railway bridges due to misalignment and defects of tracks. However, this is not currently considered an issue in UHS trains, as they float inside a vacuum tube with minimum friction. Therefore, the main parameters affecting the DAF in UHS is the resonance response of the structural system to UHS dynamic loading.

The theoretical and experimental study [12] elegantly transforms the problem of a moving force into the frequency domain however this cannot easily account for changes in system mass with time. Latterly many authors [13–17] seek to explore the dynamics of the bridge and sprung mass dynamics of the HS trains using finite element type formulations and experimental evidences. While these studies are important they are focused on very clearly defined engineering problems of specific trainsets traveling in relatively low speeds, across well-defined bridges. Thus, the problem of the dynamics of ultra-high-speed trains (UHS trains), such as the Hyperloop/Transpod, are unexplored to date.

The aim of this paper is to explore the likely envelope in the dynamic behaviour of these Hyperloop/Transpod UHS trains passing across continuous, multi-span, bridges of any span length. We derive the system equations of motion for this problem, using a "tensorial" Rayleigh-Ritz type formulation. After identifying all the key non-dimensional groups, we perform a systematic parametric exploration of this problem. Finally, we propose a likely upper bound to the dynamic amplification factor (DAF) imposed on this class of bridges for a generic class of UHS trains. Furthermore, we seek to determine whether the current design code recommendations are suitable for such UHS trains.

## 2. Theory

In this section we derive, from first principles, the equations of motion of a train composed of a set of moving masses/loads traveling at any speed across a continuous beam of any span length with any number of spans. Consider a set $k$ moving masses $m_p$ (at positions $x_p$) traveling at some group velocity $v$ across a continuous beam of $n$ spans of length $L$; shown in the Fig. 2. The beam has a uniform mass per unit length $m$ and flexural rigidity $EI$.

### 2.1. Lagrangian formulation

The kinetic energy $Q$ for this system is composed of two terms; (i) is due to the kinetic energy of the bridge and (ii) is due to the kinetic
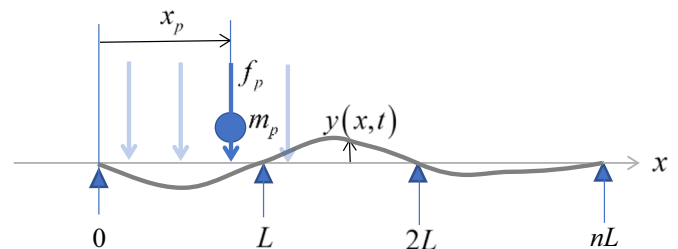


**Fig. 2.** A train composed of set of $k$ moving point masses traveling across an $n$ span continuous beam.

energy of the train. $Q$ is defined as follows

$$Q = \frac{1}{2}m \int_0^{nL} \dot{y}^2 \mathrm{d}x + \frac{1}{2} \sum_{p=1}^{k} \{m_p \dot{y}(x_p, t)^2 \beta(x_p)\}, \tag{1}$$

where $y(x,t)$ is the spatiotemporal beam displacement. In UHS trains, the moving masses $m_i$ would correspond to the locations of the magnetic levitation bearings. In conventional HS trains, this would correspond to the wheelset locations. The boxcar function $\beta(x)$ ensures only traveling masses "on the beam" are included in the energy considerations. This boxcar function can be defined as follows in terms of Heaviside functions $H(x)$ thus,

$$\beta(x) = H(x) - H(x - nL) \tag{2}$$

To simplify the resulting equations of motion and to identify all the key non-dimensional parameter groups we introduce a non-dimensional coordinate $\xi$ where $x = \xi L$ and train moving mass positions are $x_p = \xi_p L$. Hence, Eq. (1) is re-stated as follows,

$$Q = \frac{mL}{2} \int_0^{n} \dot{y}^2 \mathrm{d}\xi + \frac{1}{2} \sum_{p=1}^{k} \{m_p \dot{y}(\xi_p, t)^2 \beta(\xi_p)\}, \tag{3}$$

where the beam displacement is now $y(\xi, t)$. Note that this change in coordinates $x = \xi L$ changes the integral limits to 0 to $n$.

The potential energy $V$ of the system is also composed of two terms; (i) is the internal flexural strain energy of the beam and (ii) is external work done in moving the gravitational load of the train. $V$ is defined as follows

$$V = \frac{1}{2}EI \int_0^{nL} \left(\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}\right)^2 \mathrm{d}x - \sum_{p=1}^{k} \{f_p y(x_p, t)\beta(x_p)\}, \tag{4}$$

where the gravitational train loads are $f_p = -m_p g$. As before we introduce non-dimensional coordinate $\xi$ where $x = \xi L$. Hence, beam curvature is redefined as follows, $\mathrm{d}^2 y/\mathrm{d}x^2 = (1/L^2)\mathrm{d}^2 y/\mathrm{d}\xi^2$. We use the Newtonian prime notation $(\cdot)'' = \mathrm{d}^2 y/\mathrm{d}\xi^2$ and hence Eq. (4) is re-expressed as