Structures 14 (2018) 164–177

Contents lists available at ScienceDirect

Structures

journal homepage: www.elsevier.com/locate/structures

An Interactive Solution for Lateral-Torsional Buckling of the Mono-Symmetric Beam-Columns with Discrete Lateral Bracings

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ARTICLE INFO

Keywords:
Mono-symmetric I-sections
Steel beam-columns
Buckling strength
Bracing stiffness requirement
Finite element method

ABSTRACT

The beam-column members are widely used in civil structures. These members are subjected simultaneously to the axial load and the bending moment. In this paper, the buckling strength and the bracing stiffness requirement of the mono-symmetric beam-columns with discrete lateral bracing have been investigated. For this purpose, a numerical method based on the finite element analysis is developed to calculate the buckling load-set of the mono-symmetric beam-columns. This proposed method is capable to consider the initial geometric imperfection. The proposed method is compared with the results of experimental and numerical studies obtained by previous researchers and the spatial finite element analysis, which it shows good agreement. Also, by using the matrix form of the energy equation, a closed form solution of the bracing stiffness requirement is suggested for an arbitrary number of bracing points. The P-M interaction curve and formula are presented for mono-symmetric beam-columns. The results show that the peak point of the curve occurs at the $P = P_{ey}$ and $M = P_{ey}y_0$, where P_{ey} is Euler's buckling load. Finally, a parametric study is done to investigate the effects of the eccentricity and degree of mono-symmetry on the buckling strength and bracing stiffness requirement.

1. Introduction

Beam-column refers to a structural member that is subjected simultaneously to the axial load and the bending moment [1]. In beam-columns, both bending moment and axial load interactively affect the buckling behavior. This effect depends on the boundary and loading conditions. Lateral bracings can be increased the buckling strength of the beam-columns. Furthermore, the buckling behavior of the mono-symmetric sections is different from the doubly symmetric sections. Therefore, study of the buckling behavior of laterally braced mono-symmetric beam-columns gives some thoughts on the stability criteria.

Winter [2], Galambos [3], Timoshenko and Gere [4], Pincus [5] and Trahair [6] had done oldest studies on behavior of beam-columns. Plaut [7] studied the buckling of columns with lateral bracing at arbitrary point. He suggested an equation to predict bracing stiffness requirement. This work was continued by Plaut and Yang [8] for columns with three equal or unequal spans. Gosowski [9] investigated the buckling behavior of mono-symmetric beam-columns with bracings using differential equations and experimental studies. His analytical solution is suitable for beam-columns with arbitrary boundary and loading conditions. The effects of the strength and stiffness of the beam bracing have been studied by Yura [10]. He confirmed that the lateral bracing is most efficient than the torsional bracing to conditions such as top flange loading. Goncalves and Comotim [11] studied the beam-column interaction formula for various loading and boundary conditions, using finite element method. Their paper presents an accurate solution for calculating equivalent moment factor, C_m. McCann et al. [12] studied the stability of discretely braced beams using Rayleigh-Ritz analysis. They investigated sequential critical mode progression using harmonic representation.

In this paper the buckling behavior of the mono-symmetric beam-columns with discrete lateral bracings is investigated and the bracing stiffness requirement is presented. The beam-columns are simply supported and subjected to the pure bending moment and constant axial load. 1D finite element method (FEM) is developed to calculate the buckling load-set of mono-symmetric beam-columns. This finite element (FE) formulation includes the effects of the elastic bracing stiffness and initial geometric imperfection. Using MATLAB [13], a computer program has been written which it can calculate the buckling load-set of unrestrained, restrained and imperfect beam-columns. The proposed FEM program is verified with the results of previous experimental and analytical studies and also the 3D FE analyses. 3D FE analyses are done using Abaqus [14] software. Subsequently, the closed form solution of the bracing stiffness requirement is proposed, using the matrix form of the energy equation and the interaction P-M curve is presented. Also a nonlinear summation of the utilization ratios of the

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https://doi.org/10.1016/j.istruc.2018.03.006
Received 21 September 2017; Received in revised form 15 December 2017; Accepted 17 March 2018
Available online 20 March 2018

stresses is proposed as a design criteria. Finally, a parametric study is done to investigate the effects of the eccentricity and degree of mono-symmetry on the buckling strength and bracing stiffness requirement.

Nomenclature

A	Cross-sectional area
E	Young's modulus
G	Shear modulus
$[G]$	Geometric or stability matrix
I_t	Torsional constant
I_w	Warping constant
I_x	Moment of inertia about the strong axis
I_y	Moment of inertia about the weak axis
$I_{y,top}$, $I_{y,bot}$	Top and Bottom flange moments of inertia
I_{cf}	Compression flange moment of inertia
$[K]$	Out-of-plane stiffness matrix
$\overline{K}$	Beam parameter
L	Length of the beam-column
M	Applied bending moment
M_e	Pure bending buckling capacity
M_n	Full braced bending capacity
P	Applied axial load
P_e	Pure axial buckling capacity
P_n	Full braced axial capacity
$[R]$	Restraints matrix
U	Strain energy
V	Potential energy of applied loads
d'	Distance between flange centroids
e	$=$M/P eccentricity
n	Number of bracing points
m	Number of buckling mode
${r_d}^2$	$=(I_x + I_y)/A$, Polar radius of gyration
${r_m}^2$	$={r_d}^2 + {y_0}^2$
u,v, w and φ	Buckling deformations
x,y and z	Principal centroid axes
y_0	Coordinate of the shear center
y_R	Vertical distance between the bracing point and shear center
α_L	Lateral bracing stiffness
α_{LT}	Lateral bracing stiffness requirement
β_x	Mono-symmetric property
δ	Deformations vector
μ	Eigenvalue
ρ	Degree of mono-symmetry
$[\Phi]$	Shape function matrix
ψ	Amplitude of initial geometric imperfection

2. Buckling properties of mono-symmetric I-sections

For mono-symmetric I-sections, since the moment of inertias of flanges are not equal, the points of centroid and shear center do not coincide and distance between them can be calculated by degree of mono-symmetry ρ, as given by

$$\rho = \frac{I_{cf}}{I_{ytop} + I_{ybot}} \approx \frac{I_{cf}}{I_y} \tag{1}$$

According to Eq. (1), when the compression flange is larger than the tension flange, ρ is larger than 0.5, when the compression flange is smaller than the tension flange, ρ is smaller than 0.5, and for doubly symmetric I-sections, ρ equals to 0.5 (see Fig. 1).

When a doubly-symmetric I-section is under the pure compression load, the first buckling mode is lateral buckling and this element only deflects out-of-plane. For mono-symmetric I-sections, in same loading condition, out-of-plane deflection and twisting occur simultaneously and the first buckling mode is the lateral-torsional buckling. This

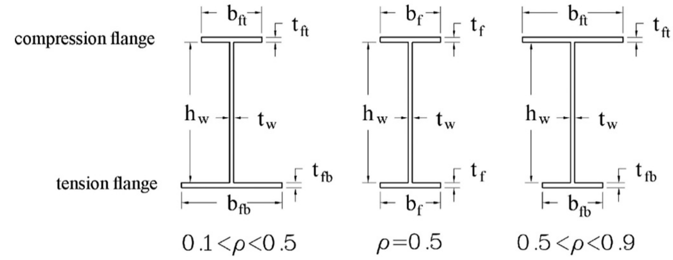

Fig. 1. Dimensional notations and degree of mono-symmetry.

different behavior is caused by an additional torque that known as Wagner effect [15]. In mono-symmetric columns, this additional torque reduces the torsional resistance of section from $GI_t\varphi$ to $(GI_t - Pr_0)\varphi$, [16]. Also, the torsional resistance of mono-symmetric beams changes from $GI_t\varphi$ to $(GI_t - M\beta_x)\varphi$, [17]. Kitipornchai and Trahair [18] proposed a simplified equation for mono-symmetry property β_x as

$$\beta_x = 0.9d'(2\rho - 1)\left[1 - \left(\frac{I_y}{I_x}\right)^2\right]. \tag{2}$$

3. Lateral-torsional buckling analyses

3.1. General analytical equations of buckling strength of mono-symmetric sections

An axially loaded column with doubly-symmetric cross-section may buckle laterally or torsionally, depending on the bending and torsion resistances. Therefore, the critical buckling load of doubly-symmetric column is equal to the lowest of flexural and torsional buckling loads [19], as given by

$$P_{ey} = \frac{\pi^2 EI_y}{L^2} \tag{3}$$

$$P_{ez} = \frac{1}{r_d^2}\left(GI_t + \frac{\pi^2 EI_w}{L^2}\right) \tag{4}$$

Due to the imbalance of mono-symmetric I-sections, these are buckled into the lateral or lateral-torsional buckling mode. Therefore, the critical buckling load (also known as pure axial buckling capacity) of mono-symmetric columns is equal to the lateral-torsional buckling load, as stated in [19], as follows:

$$P_e = \frac{(P_{ey} + P_{ez}) \pm \sqrt{\left\{(P_{ey} + P_{ez})^2 - 4P_{ey}P_{ez}\, r_d^2/r_m^2\right\}}}{2r_d^2/r_m^2} \tag{5}$$

The lateral-torsional buckling moment (also known as pure bending buckling capacity) of the mono-symmetric I-beams can be calculated by

$$M_e = \frac{\pi^2 EI_y\beta_x}{2L^2}\left\{1 \pm \sqrt{1 + \frac{4}{{\beta_x}^2}\left[\frac{I_w}{I_y} + \frac{GI_t L^2}{\pi^2 EI_y}\right]}\right\}. \tag{6}$$

3.2. Energy equation

For an ideal structure, when the total potential energy, for any small displacement, remains constant, the equilibrium position of structure is neutral [20]. The total potential energy is obtained by summing the strain energy and the potential energy of the loads. In this paper, the bifurcation buckling load of beam-columns is derived based on the energy approach.

As shown in Fig. 2, the longitudinal axis is designated by z and the strong and weak axes of section are designated by x and y, respectively. The corresponding deformations to the x, y and z axes are considered as u, v and w, respectively. Also, the out-of-plane rotation is designated by φ.