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Nikolaos I. Ioakimidis

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Nikolaos I. Ioakimidis

School of Engineering, University of Patras, GR-265 04 Patras, Greece

Abstract

The classical energy method for the approximate determination of critical buckling loads of bars is revisited. This method is based on the stability condition of the bar and on the appropriate selection of an approximation to the deflection of the bar. Moreover, it is frequently related to the Rayleigh quotient or to the Timoshenko quotient for the determination of the critical buckling load. Here we will use again the energy method for the determination of critical buckling loads of bars but now on the basis of a new computational approach. This new approach consists in using the modern computational method of quantifier elimination efficiently implemented in the computer algebra system *Mathematica* instead of partial differentiations when we use the stability condition of the bar or essentially equivalently when we minimize the Rayleigh quotient or the Timoshenko quotient. This approach, which avoids partial differentiations, is also more rigorous than the classical approach based on partial derivatives because it does not require the use of the conditions for a minimum based on second partial derivatives, which are generally ignored in practice. Moreover, it is very simple to use inside the powerful computational environment offered by *Mathematica*. The present approach is illustrated in several buckling problems of bars including parametric buckling problems. Buckling problems of bars with two internal unilateral constraints, where the classical energy method is difficult to apply, are also studied. Even in this rather difficult application the critical buckling load is directly determined with a sufficient accuracy.

Keywords: Buckling, Bars, Columns, Critical buckling load, Energy method, Rayleigh quotient, Timoshenko quotient, Internal roller supports, Bilateral and unilateral constraints, Parametric buckling problems

1. Introduction

The energy method is a very well-known and efficient approximate method for the determination of critical buckling loads of bars (or almost equivalently columns or even beam–columns) devised for this class of problems by Timoshenko; see e.g. [1, pp. 82–98], [2, Chap. 5, pp. 305–369] and [3, Chap. 5, pp. 145–171]. This method is based on the classical stability condition of the bar, $\Delta U > \Delta W$, which uses the increase ΔU of the strain energy of the bar during buckling due to its deflection $y(x)$ and the work ΔW done by the buckling load P. Moreover, it is based on an assumed appropriate approximation $y_n(x)$ to the deflection $y(x)$ of the bar, which includes one or frequently more than one parameter.

The energy method leads to the determination of the critical buckling load through a quotient which must be minimized with respect to the parameter(s) involved. This quotient can be either the Rayleigh quotient or the Timoshenko quotient [2, pp. 323–339], [3, pp. 149–152]. Both the Rayleigh and the Timoshenko quotients lead to upper bounds of the critical buckling load but the Timoshenko quotient leads to a lower and therefore more accurate upper bound. This happens since the Rayleigh quotient uses both the first and the second derivatives of the approximate deflection $y_n(x)$ of the bar whereas the Timoshenko quotient uses only the first derivative of this deflection as well the same deflection itself. In this way, the second derivative, which includes a further error because of the second differentiation of $y_n(x)$ is avoided. On the other hand, the Rayleigh quotient is more general in its applicability compared to the Timoshenko quotient. Of course, the use of both of these quotients can be avoided by directly using the aforementioned stability condition $\Delta U > \Delta W$ and this essentially constitutes an application of the Rayleigh–Ritz method (or Ritz method) [3, pp. 156–160] to the present buckling problem for a bar (or almost equivalently a column or a beam–column).

The classical approach in the energy method for the determination of critical buckling loads of bars consists in the minimization of the related quantity (here a functional), i.e. the Rayleigh quotient or the Timoshenko quotient, or alternatively in the direct use of the appropriate total energy increase $\Delta\Pi = \Delta U - \Delta W$, by reducing it to a system of linear algebraic equations with respect to the parameter(s) involved in the adopted approximation $y_n(x)$ to the deflection $y(x)$ of the bar. Naturally, the minimization of the Rayleigh quotient or the Timoshenko quotient takes place through the computation of the first derivatives of these quotients, which obviously should be set equal to zero. As is extremely well-known from differential calculus, this is a necessary condition for the existence of a minimum of a function of one variable or of more than one variable. But on the other hand, this is not also a suffi-

*Correspondence to: 8 Polyzoidi street, GR-264 42 Patras, Greece. Tel.: +30 2610 432257.

Email address: n.ioakimidis@upatras.gr (Nikolaos I. Ioakimidis)