Thin-Walled Structures 129 (2018) 1–9

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Full length article

Accurate thermal buckling analysis of functionally graded orthotropic cylindrical shells under the symplectic framework

Yiwen Ni, Zhenzhen Tong, Dalun Rong, Zhenhuan Zhou*, Xinsheng Xu

State Key Laboratory of Structure Analysis of Industrial Equipment and Department of Engineering Mechanics, Dalian University of Technology, International Center for Computational Mechanics, Dalian 116024, PR China

ARTICLE INFO

Keywords:
Hamiltonian system
Symplectic method
Thermal buckling
Analytical solution
Functionally graded material
Cylindrical shell

ABSTRACT

Accurate thermal buckling analysis of functionally graded orthotropic cylindrical shells is presented based on the Reissner's shell theory under the symplectic framework. By introducing a full-state vector, the high-order governing differential equation is reduced into a set of low-order ordinary differential equations. The fundamental unknowns are expanded in terms of the symplectic eigensolutions without any trial function. The buckling equations and buckling mode shapes are analytically obtained. The present study demonstrates that the expressions of displacements have different forms and strongly depend on the end conditions and thickness. Some new results are presented in numerical examples.

1. Introduction

The applications of functionally graded materials (FGMs) have attracted much attention in recent years especially in extreme high temperature environments. Typically, the FGMs are made of a mixture of ceramic and metal. The ceramic component provides high temperature resistance while the ductile metal component prevents fracture. Nowadays, due to the excellent carrying capacity of the shell-like structure, functionally graded (FG) cylindrical shells are increasingly being used in the development of high-speed spacecrafts, nuclear fusion reactors, thermo-generators, etc [1,2]. Therefore, the thermal stability of such shells under high temperature conditions is very essential.

Thermal buckling of cylindrical shells has been well established in the field of structural stability [3–6]. The literature available on the elastic cylindrical shell is much more extensive than for the FG cylindrical shells. In the study of thermal buckling of FG cylindrical shells, Shahsiah and Eslami [7,8], Wu et al. [9], Yaghoobi et al. [10] and Bagherizadeh et al. [11] obtained the closed-form solutions for thermal buckling of FG cylindrical shells. Sheng and Wang [12] analyzed the thermal buckling and dynamic stability of FG cylindrical shells under thermal and mechanical loads. Shariyat [13] studied the dynamic thermal buckling of FG cylindrical shells with sudden heat. Mirzavand and his collaborators [14–16] found exact solutions for thermal buckling of imperfect FG cylindrical shells. Shen [17–20] obtained exact solutions for thermal buckling and postbuckling of FG cylindrical shells. Kiani and his collaborators [21–25] derived exact solutions for thermal buckling of FG cylindrical and conical shells. More recently, Sofiyev et al. [26] and Duc et al. [27] studied thermal buckling of FG conical shells.

In view of the aforementioned literature, the buckling of FG cylindrical shells was reduced to a high-order governing partial differential equation in the classical system [9,15] which were solved by some pre-determined functions; most of the works were accomplished based on the simplified shell (e.g., Donnell's shell theory). The simplified assumptions and trial functions simplify the solution procedure but only provide the approximate solution in most cases. An accurate cylindrical shell model will provide more reasonable guidance to the design of the thermal barrier structures. This paper aims to develop an accurate thermal buckling model of FG orthotropic cylindrical shells based on the Reissner's shell theory. Exact solutions for thermal buckling of FG orthotropic cylindrical shells will be obtained by a Hamiltonian-based method [28–34].

The paper is organized in the following way. Firstly, the high-order governing differential equation is reduced to a set of low-order ordinary differential equations in the Hamiltonian system. Then, analytical thermal buckling equations and buckling mode shape functions are obtained. Finally, comparisons and numerical examples are provided.

2. Material properties of FG cylindrical shells

A FG orthotropic cylindrical shell subjected to a temperature rise ΔT, with length L, radius R and thickness h is shown in Fig. 1. The cylindrical shell is referred to a coordinate system (x, θ, z) in which x and θ are in the axial and circumferential directions of the shell and z is

* Corresponding author.
E-mail address: zhouzh@dlut.edu.cn (Z. Zhou).

https://doi.org/10.1016/j.tws.2018.03.030
Received 26 December 2017; Received in revised form 14 March 2018; Accepted 27 March 2018

Nomenclature

E, υ, α	Young's modulus, Poisson's ratio and thermal expansion coefficient
L, h, R	Axial length, thickness and middle radius
N	Power-law exponent
$\mathbf{H}$	Hamiltonian operator matrix
$\boldsymbol{\psi}$	Total unknown vector
$\boldsymbol{\varphi}$, $\boldsymbol{\gamma}$	Resultant forces and moments caused by a temperature change
$\mathbf{q}$, $\mathbf{p}$	Original vector and dual vector
μ, $\boldsymbol{\eta}$	Symplectic eigenvalue and Symplectic eigenfunction
θ_θ	Angle of rotation
A_{ij}, B_{ij}, D_{ij}	Extensional, coupling and bending stiffnesses
L_C	Lagrangian density function
Q_{kl}	Reduced stiffness
T_0	Initial temperature
ΔT_{cr}	Critical thermal buckling load
V_x, V_θ	Equivalent shear forces

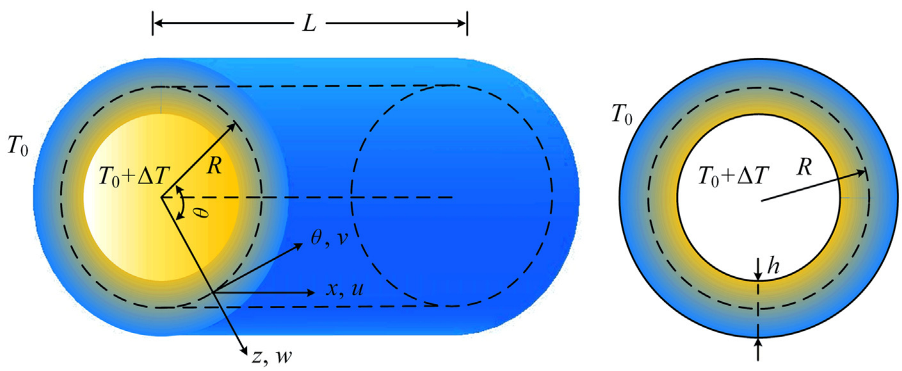

Fig. 1. Geometry of a FG orthotropic cylindrical shell with thermal load.

in the direction of the outward normal to the middle surface. The FG cylindrical shell is assumed to be made of two different material components and the material properties vary continuously in the thickness direction [7,15]. The effective material properties can be expressed as

$$P_x(T,\ \ z) = [P_{xo}(T) - P_{xi}(T)]\left(\frac{z + h/2}{h}\right)^N + P_{xi}(T), \tag{1a}$$

$$P_\theta(T,\ \ z) = [P_{\theta o}(T) - P_{\theta i}(T)]\left(\frac{z + h/2}{h}\right)^N + P_{\theta i}(T) \tag{1b}$$

where P may be used to substitute Young's modulus E, Poisson's ratio υ and thermal expansion coefficient α; the subscripts "o" and "i" denote the outer and inner material components, respectively; N stands for the power-law index; T is the ambient temperature (in Kelvin). Here, it should be stated that the material properties of outer and inner surfaces of the shell (P_{ko} and P_{ki}, $k = x, \theta$) can be temperature dependent and the dependency on temperature may be expressed in terms of the following higher order Touloukian representation [35], i.e., $P_{kl}(T) = P_0(P_{-1}T^{-1} + 1 + P_1 T + P_2 T^2 + P_3 T^3)$ where $k = x,\ \theta,\ l = i,\ o$ and P_0, P_{-1}, P_1, P_2 and P_3 are the coefficients of temperature T.

3. Basic equations of FG cylindrical shells

On the basis of Reissner's shell theory [36], the displacements along x-, θ - and z- axes are specified by u, v and w, respectively. The strain vector $\mathbf{e} = \{e_x,\ \ e_\theta,\ \ e_{x\theta}\}$ can be expressed as

$$\mathbf{e} = \boldsymbol{\varepsilon} + z\boldsymbol{\chi} \tag{2}$$

where

$$\boldsymbol{\varepsilon} = \{\varepsilon_x,\ \ \varepsilon_\theta,\ \ \varepsilon_{x\theta}\} = \left\{\frac{\partial u}{\partial x},\ \ \frac{\partial v}{R\partial\theta} + \frac{w}{R},\ \ \frac{\partial v}{\partial x} + \frac{\partial u}{R\partial\theta}\right\} \tag{3}$$

and

$$\boldsymbol{\chi} = \{\chi_x,\ \ \chi_\theta,\ \ \chi_{x\theta}\} = \left\{-\frac{\partial^2 w}{\partial x^2},\ \ -\frac{1}{R^2}\left(\frac{\partial^2 w}{\partial\theta^2} - \frac{\partial v}{\partial\theta}\right),\ \ -\frac{1}{R}\left(2\frac{\partial^2 w}{\partial\theta\partial x} - \frac{\partial v}{\partial x}\right)\right\} \tag{4}$$

are the strain and curvature vectors at the shell middle surface, respectively.

The temperature rise of the cylindrical shell in the ambient temperature is assumed to be $T(z) = \Delta T$. The constitutive relation with thermal effects [37,38] is given by

$$\boldsymbol{\sigma} = \mathbf{Q}\cdot\mathbf{e} - \boldsymbol{\sigma}_T \tag{5}$$

where $\boldsymbol{\sigma} = \{\sigma_x,\ \ \sigma_\theta,\ \ \sigma_{x\theta}\}$ is the stress vector; $\boldsymbol{\sigma}_T = \{\sigma_{Tx},\ \ \sigma_{T\theta},\ \ 0\}$ is the thermal stress vector, $\sigma_{Tx} = Q_{11}\alpha_x T + Q_{12}\alpha_\theta T$, $\sigma_{T\theta} = Q_{21}\alpha_x T + Q_{22}\alpha_\theta T$; $\mathbf{Q}$ is the stiffness matrix and its non-zero components are $Q_{11} = E_x/(1 - \upsilon_x\upsilon_\theta)$, $Q_{22} = E_\theta/(1 - \upsilon_x\upsilon_\theta)$, $Q_{12} = \upsilon_\theta E_x/(1 - \upsilon_x\upsilon_\theta)$, $Q_{21} = \upsilon_x E_\theta/(1 - \upsilon_x\upsilon_\theta)$, $Q_{66} = E_\theta/[2(1 + \upsilon_x)]$; $\upsilon_x E_\theta = \upsilon_\theta E_x$.

The resultant force vector $\mathbf{N} = \{N_x,\ \ N_\theta,\ \ N_{x\theta}\}^{\mathrm{T}}$ and moment vector $\mathbf{M} = \{M_x,\ \ M_\theta,\ \ M_{x\theta}\}^{\mathrm{T}}$ of the cylindrical shell are given by

$$\{\mathbf{N},\ \ \mathbf{M}\} = \int_{-h/2}^{h/2} \boldsymbol{\sigma}\cdot\{1,\ \ z\}dz. \tag{6}$$

Substituting Eqs. (2) and (5) into Eq. (6), the internal forces are expressed in terms of mid-surface displacements, i.e.,

$$\begin{Bmatrix}\mathbf{N}\\ \mathbf{M}\end{Bmatrix} = \begin{bmatrix}\mathbf{A} & \mathbf{B}\\ \mathbf{B} & \mathbf{D}\end{bmatrix}\begin{Bmatrix}\boldsymbol{\varepsilon}\\ \boldsymbol{\chi}\end{Bmatrix} - \begin{Bmatrix}\boldsymbol{\varphi}\\ \boldsymbol{\gamma}\end{Bmatrix} \tag{7}$$

where $\{A_{ij},\ \ B_{ij},\ \ D_{ij}\} = \int_{-h/2}^{h/2} Q_{ij}\{1,\ \ z,\ \ z^2\}dz$ (i, $j = 1$, 2 and 6) are components of the extensional stiffness matrix $\mathbf{A}$, coupling stiffness matrix $\mathbf{B}$ and bending stiffness matrix $\mathbf{D}$, $A_{12} = A_{21}$, $B_{12} = B_{21}$ and $D_{12} = D_{21}$; $\boldsymbol{\varphi} = \int_{-h/2}^{h/2}\boldsymbol{\sigma}_T dz$ and $\boldsymbol{\gamma} = \int_{-h/2}^{h/2}\boldsymbol{\sigma}_T z dz$ indicate the resultant forces and moments caused by the temperature rise ΔT, respectively.

Consider pre-buckling axisymmetric deformations before buckling occurs. The displacements and stress resultants are functions of x-coordinate only. Hence, specifying the corresponding quantities with superscript "0", the bucking is mainly caused by an increase of internal forces $N_x^0 = N_T^0$, $N_\theta^0 = 0$ and $N_{x\theta}^0 = 0$ [9,39] where $N_T^0 = \int_{-h/2}^{h/2}[(E_x\alpha_x + \upsilon_\theta E_x\alpha_\theta)/(1 - \upsilon_x\upsilon_\theta)]Tdz$ represents the internal forces caused by the thermal loads. Here, the pre-buckling force is taken as $N_x^0 = N_T^0$ because the surfaces of the FG cylindrical shell are free [4,5,7–9,17,31,32]. Specifying the internal forces and displacements with superscript "1" and retaining the linear terms [40], the governing