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B-spline finite element approach for the analysis of thin-walled beam structures based on 1D refined theories using Carrera unified formulation

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ABSTRACT

In the current study, 1D refined beam theories on the basis of Carrera Unified Formulation (CUF) are combined with Isogeometric approach (IGA) for the static and free vibration analysis of thin-walled beam structures. The B-spline basis functions utilized in IGA are employed to approximate displacement field due to their interesting attributes in analysis. *N-Order* Taylor-like expansions are utilized in the framework of CUF which presents finite element matrices in the form of fundamental nuclei that is independent of the type and order of expansion. Higher-order B-spline basis functions attenuate the effect of shear locking properly and higher-order theories presented by CUF are free from Poisson locking phenomenon and utilizations of shear correction factor. Several numerical results including both solid and thin-walled structures are investigated and it is shown that coupling IGA and CUF ends in a suitable methodology to analyze beam structures.

1. Introduction

Analysis of slender bodies is a major concern in the design of aerospace, mechanical, and civil structures. In fact, beam structures are widely used as aircraft wings, helicopter rotor blades in aerospace engineering or as bridge decks in civil engineering industry. Hence, understanding the manner of beams under different conditions such as bending and free vibration seems to be crucial.

Classical beam theories were first introduced by Euler-Bernoulli [1] and then by Timoshenko [2,3]. Euler-Bernoulli theory assumes that the cross section of the beam is infinitely rigid in its own plane due to neglecting transverse strains. To overcome this limitation, Timoshenko accounts for shear deformations and also the cross section does not remain normal after deformation of the beam, albeit it is still rigid on its plane. As a legacy of mentioned assumptions, the classical models cannot capture the higher-order manner of beam structures such as warping, in-plane distortion of cross section and shear effects. To compensate these drawbacks of classical theories, various higher-order shear deformation theories (HSDTs) have been developed and some of them are well documented in [4,5]. HSDTs are usually formulated by axiomatic assumptions and based on fixed-order expansions of the generalized unknowns. Besides eliminating Poisson locking and the need for shear correction factors, these theories provide more realistic mathematical models to consider higher-order effects. In addition, the displacement field expansions employed in CUF, are able to expand unknown fields with any order of expansion. Indeed, the order of expansion is considered as a free parameter in CUF. Carrera introduced a class of 2D theories using a compact notation in [6] which was later named as CUF in literature. Mentioned compact notation facilitates expanding displacement field to an arbitrary order of expansion. To this end, a single 3×3 matrix so-called "*fundamental nuclei*" has been utilized that can easily formulate variable description without any dependency on type and order of expansion. The theoretical foundation of the unified method is presented in the comprehensive article by Carrera [7]. Then, the CUF has been developed in the formulation of beam structures [8–15].

Besides higher-order theories, IGA is employed to compensate for the drawbacks of classical finite element methods. During the past decades, the finite element method has been developed as a strong tool for engineering problems, whereas a discretized geometry obtained through the meshing process is needed which often results in geometrical errors. In addition, to reach the desired accuracy, a remeshing process is required during analysis that leads to a time-consuming process because of the interaction between the CAD system and the analysis. Hence, the initial work in Isogeometric analysis was motivated by the existing gap between the worlds of finite element analysis (FEA) and computer-aided design (CAD). In fact, IGA has overcome this gap by using the same functions to describe the exact geometry and approximate the solution field. This means that the mesh refinement is simply accomplished by reindexing the parametric space. Indeed, the refinement process can proceed without interaction with the CAD system [16,17]. NURBS (non-uniform rational B-spline) functions as a

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general form of B-spline functions were first used by Hughes et al. [16] as basis functions to approximate solution field. Furthermore, NURBS-based approaches have been developed in a wide range of research areas such as fluid–structure interaction [18–20] shell analysis [21–25], structural analysis [26,27], fracture mechanics [28,29]. Also, a combination of IGA and CUF has been proposed to analyze composite laminated plates, see [30,31]. Furthermore, a numerical overview of IGA has been presented in [32].

In this paper, B-spline functions are adopted along with CUF, whose hierarchical capabilities allow one to adopt different approximation function indistinctly. B-spline functions are employed in this article due to their interesting attributes. Besides a precise geometric modeling, the B-spline functions show unique properties in analysis. B-spline functions not only depict the exact geometry of the beam model, but also they show unique characteristics in analysis. The order of the B-spline functions can be applied as a free parameter in analysis and it can be counted as one of their most obvious features. Given this specification, higher-order B-spline functions can be utilized to reduce the effect of shear locking phenomenon.

2. Refined beam theories by CUF

As it is mentioned previously, classical beam models cannot foresee higher-order manner of beam structures. Therefore, more sophisticated models are needed to predict higher-order variables [33]. CUF defines the displacement field in a compact form [10] as in Eq. (1):

$$\boldsymbol{u}(x, y, z) = F_\tau(x, z)\boldsymbol{u}_\tau(y), \qquad \tau = 1, 2, \ldots, M \tag{1}$$

Where $\boldsymbol{u}$ *(x, y, z)* is the 3D displacement vector, F_τ are generic functions on the cross section in the coordinates x and z. $\boldsymbol{u}_\tau$ is the generalized displacement vector along the beam axis. Also, τ represents summation and M stands for the number of terms used in the expansion. Using a Taylor-like expansion, the displacement field can be expanded to an arbitrary order. For instance, a linear expansion ($N = 1$) of solution field can be developed as follows:

$$\begin{aligned} u_x &= u_{x_1} + x u_{x_2} + z u_{x_3} \\ u_y &= u_{y_1} + x u_{y_2} + z u_{y_3} \\ u_z &= u_{z_1} + x u_{z_2} + z u_{z_3} \end{aligned} \tag{2}$$

The linear expansion generates nine displacement variables, three constant and six linear. In a unified manner, higher orders of displacement field can be developed by increasing the number of expansion terms M. For example, a parabolic expansion of displacement field ($N = 2, M = 6$) can be written as:

$$\begin{aligned} u_x &= u_{x_1} + x u_{x_2} + z u_{x_3} + x^2 u_{x_4} + xz u_{x_5} + z^2 u_{x_6} \\ u_y &= u_{y_1} + x u_{y_2} + z u_{y_3} + x^2 u_{y_4} + xz u_{y_5} + z^2 u_{y_6} \\ u_z &= u_{z_1} + x u_{z_2} + z u_{z_3} + x^2 u_{z_4} + xz u_{z_5} + z^2 u_{z_6} \end{aligned} \tag{3}$$

The second-order beam model mentioned in Eq. (3) generates 18 displacement variables, three constant, six linear and nine parabolic. Taylor-like polynomials for any order of beam model are presented in Table 1.

Table 1
Taylor-like polynomials.

N	M	F_τ
0	1	$F_1 = 1$
1	3	$F_2 = x \quad F_3 = z$
2	6	$F_4 = x^2 \quad F_5 = xz \quad F_6 = z^2$
3	10	$F_7 = x^3 \quad F_8 = xz^2 \quad F_9 = xz^2 \quad F_{10} = z^3$
.	.	.
.	.	.
.	.	.
N	$(N+1)(N+2)/2$	$F_{(N^2+N+2)/2} = x^N \quad \ldots \quad F_{(N+1)(N+2)/2} = z^N$

3. Stiffness matrix

CUF assembles finite element matrices in the terms of fundamental nucleus. The Principle of Virtual Displacements (PVD) is used to derive the governing equations. According to the PVD, Eq. (4) is defined:

$$\delta L_{int} = \delta L_{ext} - \delta L_{ine} \tag{4}$$

Where δL_{int} stands for virtual variation of the strain energy, δL_{ext} denotes virtual variation of the work of external loads and δL_{ine} is the virtual variation of the work of inertial loads. Stress ($\boldsymbol{\sigma}$) and strain ($\boldsymbol{\varepsilon}$) components are arranged in Eq. (5):

$$\begin{aligned} \boldsymbol{\sigma} &= \{\sigma_{xx} \quad \sigma_{yy} \quad \tau_{xy} \quad \sigma_{zz} \quad \tau_{xz} \quad \tau_{yz}\} \\ \boldsymbol{\varepsilon} &= \{\varepsilon_{xx} \quad \varepsilon_{yy} \quad \gamma_{xy} \quad \varepsilon_{zz} \quad \gamma_{xz} \quad \gamma_{yz}\} \end{aligned} \tag{5}$$

Linear strain–displacement relations and Hooke's law can be written as:

$$\begin{aligned} \boldsymbol{\varepsilon} &= \boldsymbol{b}\,\boldsymbol{u} \\ \boldsymbol{\sigma} &= \boldsymbol{C}\,\boldsymbol{\varepsilon} \end{aligned} \tag{6}$$

Where $\boldsymbol{b}$ is operator matrix and $\boldsymbol{C}$ is material coefficient matrix that are available in [17]. Then, the virtual variation of the strain energy is considered as:

$$\delta L_{int} = \int_V \delta\varepsilon^T \boldsymbol{\sigma}\, dV \tag{7}$$

Displacement field and its virtual variation (denoted by δ) are approximated using B-spline basis functions as Eq. (8):

$$\begin{aligned} \boldsymbol{u}(x, y, z) &= F_\tau(x, z) R_i(y) \boldsymbol{u}_{\tau i}, \\ \delta\boldsymbol{u}(x, y, z) &= F_s(x, z) R_j(y) \delta\boldsymbol{u}_{sj}; \ \tau, s = 1, \ldots, M \ \ and \ \ i, j = 1, \ldots, Nc \end{aligned} \tag{8}$$

Where i and j denote summation over the B-spline shape functions and Nc is the number of control points. $F\tau$ and F_s are generic functions, R_i and R_j are B-spline functions and $\boldsymbol{u}_{\tau i}$ and $\delta\boldsymbol{u}_{sj}$ are nodal unknowns. According to the Eq. (8), B-spline functions approximate displacement field along the beam axis and, $F\tau$ and F_s describe displacement variables on the cross section. These two approximations are shown in Fig. 1 schematically.

Representing Eq. (8) into Eq. (7), a 3 × 3 matrix will be generated which is called *"fundamental nuclei"* (FN) of the stiffness matrix:

$$\boldsymbol{K}^{\tau sij} = \begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{bmatrix} \tag{9}$$

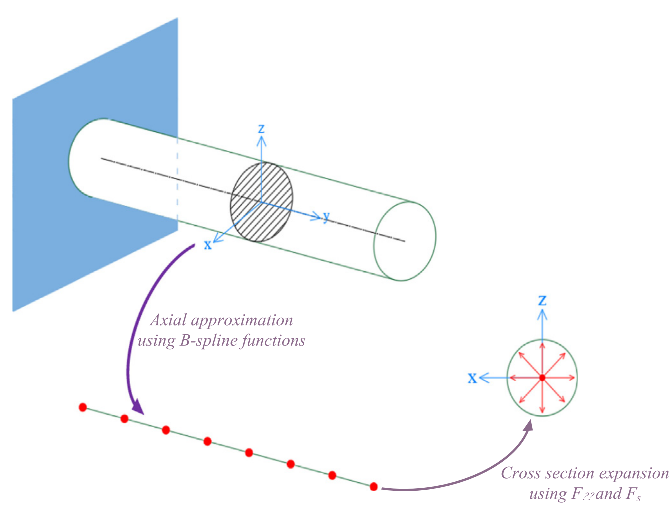

Fig. 1. Representation of the axial approximation (using B-spline basis functions), and the cross section expansion (using Fτ and F_s).