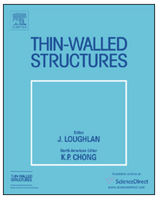
Thin-Walled Structures 129 (2018) 125–144

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws



Review

A semi-analytical method for vibration analysis of stepped doubly-curved shells of revolution with arbitrary boundary conditions

Haichao Li[a], Fuzhen Pang[a,*], Xuhong Miao[a,b], Yuan Du[a], Hongye Tian[a]

[a] College of Shipbuilding Engineering, Harbin Engineering University, Harbin 150001, PR China
[b] Naval Academy, Beijing 100161, PR China

ARTICLE INFO

Keywords:
A semi-analytical method
Jacobi polynomials
Free vibration
Stepped doubly-curved shells of revolution
Arbitrary boundary conditions

ABSTRACT

In this paper, the free vibration of the stepped doubly-curved shells of revolution is investigated by using a semi-analytical method with arbitrary boundary conditions. The stepped doubly-curved shells of revolution are divided into their segments in the meridional direction according to the steps of the structures, and the analysis of the theoretical model is formulated by using Flügge's thin shell theory. The Jacobi polynomials along the revolution axis direction and the standard Fourier series along the circumferential direction consist of the displacement functions of shell segments. The boundary conditions at the ends of the stepped doubly-curved shells of revolution and the continuity conditions at two adjacent segments were enforced by penalty method. Then, the accurate solutions about the vibration characteristic of the stepped doubly-curved shells of revolution were solved by the method of Rayleigh–Ritz. For arbitrary boundary conditions, the present method does not need any changes to the mathematical model or the displacement functions, and it is very effective in the analysis of free vibration for the stepped doubly-curved shells of revolution. The accuracy and reliability of the proposed method are verified with the results of finite element method (FEM), and some numerical results are reported for free vibration of the stepped doubly-curved shells of revolution under arbitrary boundary conditions. Results of this paper can provide reference data for future studies in related field.

1. Introduction

The stepped doubly-curved paraboloidal, elliptical, hyperbolical, circular toroidal, catenary and cycloidal shells of revolution are widely used in practical engineering applications, such as ship engineering, aerospace, chemical industry, marine and civil engineering and so on. However, due to the difficult in dealing with the boundary and the continue conditions of the doubly-curved shells of revolution, there is little research on the vibration analysis of the doubly-curved shells of revolution, expecially for the stepped doubly-curved shells of revolution. So it is very important to carry on the research about the vibration characteristics of the stepped doubly-curved shells of revolution.

Vibration analysis and numerical methods for various shell structures are described in literatures of Leissa [1], Qatu [2], and Liew et al [3]. Based on these theories, lots of researchers have proposed many analysis methods to investigate the vibration characteristic of the doubly-curved shells of revolution. Tornabene [4] analyzed the vibration characteristics of laminated composite doubly-curved shells of revolution by using the GDQ method. Naghsh et al. [5] analyzed the free vibration characteristics of stringer stiffened general revolution shells by using meridional finite strip method. Tornabene et al. [6] used GDQ (Generalized Differential Quadrature) method to obtain a 2-D solution of the free vibrations of the parabolic shells. Li et al. [7] carried on the free vibration analysis of functionally graded cylindrical, conical, spherical panels and shells of revolution with general boundary condition based on the modified Fourier–Ritz approach. Choe and Wang et al. [8] firstly proposed the method of Jacobi-Ritz to analyze the vibration of the doubly-curved shells of revolution. Pang et al. [9] developed the free and forced vibration analysis of airtight cylindrical vessels with doubly curved shells of revolution by using Jacobi–Ritz method. Xie et al. [10] presented solution approach based on the HWD (Haar Wavelet Discretization) method, and studied the free vibration of spherical and parabolic shells of revolution under arbitrary boundary conditions. Tornabene et al. [11] used the generalized displacement field of the CUF (Carrera Unified Formulation) to investigate the dynamic characteristics of doubly-curved shell of revolution. Wang et al. [12–17] used the modified Fourier series method to investigate the vibration characteristics of FGM and laminated composite doubly-curved shells of revolution. Jiang et al. [18] studied the vibration characteristic of doubly curved shallow shells with elastic edge

---

[*] Corresponding author.
E-mail address: pangfuzhen@hrbeu.edu.cn (F. Pang).

https://doi.org/10.1016/j.tws.2018.03.026
Received 1 January 2018; Received in revised form 12 March 2018; Accepted 27 March 2018

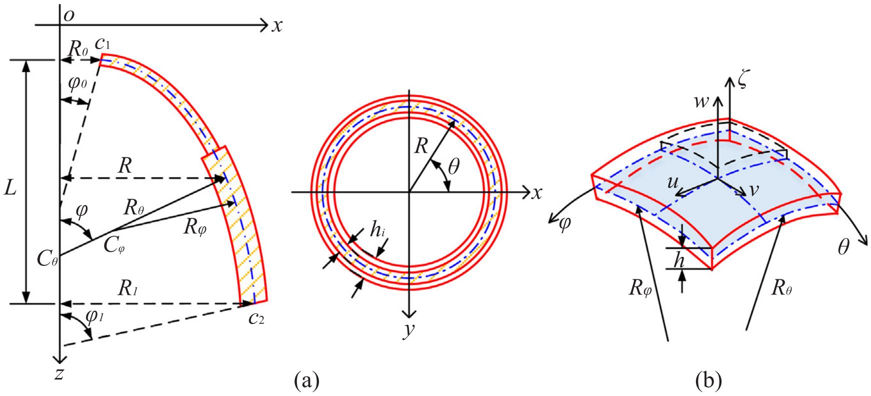


Fig. 1. Geometry notations and coordinate system of stepped doubly-curved shells of revolution: (a) cross section; (b) differential element.

restraints by the method of improved trigonometric series and Rayleigh–Ritz. Mochida et al. [19] investigated the free vibration of doubly-curved shells of revolution by using the Superposition-Galerkin method. Fadaee et al. [20] proposed a new generic exact solution for free vibration of doubly-curved shallow shells, in which a set of potential functions and auxiliary variables were used to present an exact Levy-type closed-form solution for free vibration analysis of a doubly curved FG shell panel. Guo et al. [21] proposed a kind of domain decomposition method by a modified variational method, and studied the vibration characteristics of various combination shell structures consisting of conical, cylindrical, spherical shell and ring stiffener. Tan [22] presented an efficient substructuring analysis method based on the first order shear deformation shell theory and the classical thin shell theory to investigated the free vibration characteristics for doubly-curved shells of revolution. Al-Khatib et al. [23] studied the free vibration of a closed paraboloidal shell of revolution by governing strain-displacement and curvature-displacement equations. Messina [24] studied the free vibrations of multilayered doubly-curved shells of revolution based on a mixed variational approach and global piecewise-smooth functions. Razavi et al. [25] investigated the free vibration of a simply-supported MEE doubly-curved shell resting on a Pasternak foundation based on a shear deformation theory including the rotary inertia effect. The shell is assumed to be shallow and thin so that the in-plane electric and magnetic fields can be ignored. Bacciocchi et al. [26] performed the free vibration analysis of several laminated composite doubly-curved shells, singly-curved shells and plates, characterized by a continuous thickness variation. Other literatures related to the vibration analysis of the doubly-curved shells of revolution are shown in Refs. [27–32].

For the vibration analysis of the stepped structures, Tang et al. [33] studied the free and forced vibration of multi-stepped circular cylindrical shells with arbitrary boundary conditions by the method of reverberation-ray matrix. Ganesan and Sivadas [34] investigated the free vibration of cantilever circular cylindrical shells with variable thickness. Qu et al. [35] studied the free and forced vibration analysis of uniform and stepped circular cylindrical shells using a domain decomposition method. Xie et al. [36] proposed an analytic method for free and forced vibration analysis of the stepped conical shells with arbitrary boundary conditions. Other literatures related to the vibration analysis of the stepped structures are shown in Refs. [37–40].

As we can see from the literature review, most of existing literatures are focused on the vibration analysis of the doubly-curved shells of revolution by using modified Fourier–Ritz approach, Generalized Differential Quadrature method, Haar Wavelet Discretization method, Superposition-Galerkin method, domain decomposition method, reverberation-ray matrix method and so on. And to the authors' knowledge, there are have no open literatures about the vibration analysis of the stepped doubly-curved shells of revolution. Thus, an efficient and accurate formulation is necessary and of great significance to establish to analyze the free vibration of the stepped doubly-curved shells of revolution under arbitrary boundary conditions. The aim of this paper is to present a generalized and unified Jacobi–Ritz formulation to investigate the free vibration of the stepped doubly-curved shells of revolution subjected to arbitrary boundary conditions.

The characteristic of this paper, it is reduced the calculation cost by using the penalty function method and Rayleigh–Ritz method, and as one of the advantages of the present method, it is generalized the selection of the admissible displacement functions by using the Jacobi polynomials. The stepped doubly-curved shells of revolution are divided into their segments in the meridional direction according to the steps of the structures. The displacements of all shell segments are expressed by Jacobi polynomials along the axial direction and the Fourier series along the circumferential direction. And the solutions of shell structure are calculated by using Flügge's thin shell theory and Rayleigh–Ritz method. The boundary conditions at the ends of the doubly-curved shells of revolution and the continuity conditions at two adjacent shell segments were modeled by penalty method. The natural frequencies of the stepped doubly-curved shells of revolution are decided by using the Rayleigh–Ritz method. Under the current framework, the current method is capable of handling the various stepped doubly-curved shells of revolution under arbitrary boundary conditions. The excellent convergence, accuracy, stability of the present method will be examined by several of numerical examples. In addition, some numerical examples of free vibration analysis for the stepped doubly-curved shells of revolution will conducted under arbitrary boundary conditions, which may serve as benchmark data for future researches.

2. Theoretical formulations

2.1. Description of the model

The geometry notation and coordinate system of the stepped doubly-curved shell of revolution are depicted in Fig. 1. All kinds of the stepped doubly-curved shells of revolution are made up of homogeneous and isotropic materials. The cross section of the stepped doubly-curved shells of revolution with various thickness h_i. The shell is characterized by its middle surface, which is acted as the reference surface and generated by the rotation of the meridian curve c_1c_2 about the revolution axis oz. The orthogonal coordinate system along the meridional, circumferential and normal directions of the shell is taken as φ, θ, and ζ, respectively. The symbol R_φ and R_θ describe the principal radii of curvature of the shell at its middle surface with respect to the meridional plane x-z and a plane normal to the meridian, respectively. C_φ and C_θ denote the central position of the stepped doubly-curved shells with reference to the two principal radii R_φ and R_θ. The horizontal radius R represents the distance from each point of the middle surface to the revolution axis z. Specially, it can be defined as $R = R_\theta \sin\varphi$. Fig. 1(b) shows a differential element of the shell. And the symbol